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A MONTE CARLO ERROR ANALYSIS PROGRAM FOR NEAR-MARS, FINITE-BURN, ORBITAL TRANSFER MANEUVERS

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16. Abstract

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SUMMARY

A computer program has been developed which performs an error analysis of a minimum-fuel, finite-thrust, transfer maneuver between two Keplerian orbits in the vicinity of Mars. The method of analysis is the Monte Carlo approach where each off-nominal initial orbit is targeted to the desired final orbit. The errors in the initial orbit are described by two covariance matrices of state deviations and tracking errors. The function of the program is to relate these errors to the resulting errors in the final orbit.

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INTRODUCTION

The effort of the United States to land an unmanned capsule on Mars is a very complex mission. The mission profile of Project Viking contains two or three midcourse maneuvers en route, a Mars orbit insertion maneuver, numerous orbital trim maneuvers for planetary photoreconnaissance and positioning of the spacecraft over the landing site, and finally a deorbit maneuver. The navigation and guidance problems are complicated further by the launch of a second spacecraft within a 50-day period.

For a mission of this complexity it is necessary to determine the sensitivity of the trajectory with respect to error sources which requires the establishment of the expected magnitude of the various maneuvers.

This paper deals with the error analysis of the near-Mars maneuvers (those within the sphere of influence of Mars). Of primary importance is the analysis of the Mars orbit insertion maneuver. This analysis is necessary in order to determine the orbit errors after the insertion maneuver, to calculate velocity budgets, and to determine the range of control variables for design purposes.

Several assumptions about planetary orbital insertion maneuvers have been adopted in the past to simplify the error analysis. They are as follows: (a) the maneuvers can be analyzed impulsively, (b) the encounter errors can be mapped through the maneuvers using linear propagation theory, and (c) the encounter errors can be sufficiently defined by using the impact plane and time-of-flight parameters. The validity of these assumptions is questionable for the Viking orbital insertion maneuver due to the long burn time required and the large approach errors caused by a long interplanetary trip time. Therefore, a computer program, VEAMCOP (Viking Error Analysis Monte Carlo Program), was developed to remove these assumptions from the analysis. The basic targeter for VEAMCOP is VITAP (Viking Targeting Analysis Program) given in reference 1. Although the mathematical structure of the program is quite general and therefore applicable to maneuvers near any planet, VEAMCOP was developed specifically for near-Mars maneuvers and mainly for the orbit insertion maneuver. The maneuvers considered in the analysis are minimum-fuel, finite-burn transfers between two conics. The guidance law allows for the targeting of the spacecraft to a final conic that is specified by certain constraints. In addition, the law includes an optimum maneuver in the sense that the fuel required for the transfer is a minimum.

The function of the program is to relate the errors in the initial orbit and errors in the thrusting maneuver to the resulting errors in the final orbit. The statistics of the errors in the final orbit are found by use of the Monte Carlo Method. First a sample estimate of the initial orbit is randomly generated from the statistics of the initial orbit. The random estimate is then targeted to the desired final orbit. However, due to the lack of knowledge of the actual initial orbit and due to maneuver execution errors, the final orbit will be in error. The Monte Carlo Method of repeatedly sampling the initial errors and calculating the final errors provides the statistics of the errors in the final orbit.

SYMBOLS

a	semimajor axis, kilometers
$a(t)$	magnitude of thrust acceleration at time t, kilometers/second^2

$\vec{B}$ vector from the center of the planet perpendicular to the incoming hyperbolic asymptote, kilometers

$\vec{B} \cdot \hat{R}$ targeting parameter (component of $\vec{B}$ in the $\hat{R}$ direction), kilometers

$\vec{B} \cdot \hat{T}$ targeting parameter (component of $\vec{B}$ in the $\hat{T}$ direction), kilometers

C covariance matrix

D Cartesian covariance matrix of actual state deviations from the nominal state

$E(\xi)$ expected value of ξ

e eccentricity

F point which defines the computed initial direction of thrust (see fig. 6(a))

G point which defines the actual initial direction of thrust (see fig. 6(a))

i inclination, degrees

K number of velocity counting accelerometer pulses before thrust is terminated

M dimension of vector

m mass, kilograms

m(t) mass of spacecraft at time t, kilograms

N_S number of samples

N(c,d) normal density function with mean c and variance d

$\hat{N}$ unit vector in direction of ascending node

$\hat{N},\hat{V},\hat{W}$ spacecraft coordinate system where $\hat{V}$ points in direction of velocity, $\hat{W}$ is perpendicular to the orbital plane in direction of angular momentum vector, and $\hat{N}$ completes right-handed system

$\hat{n}$	unit vector normal to plane of thrust
$\hat{P},\hat{Q},\hat{W}$	coordinate system (see fig. 6(b))
p,q,w	coordinates in $\hat{P},\hat{Q},\hat{W}$-system
R	random number from normal distribution with mean 0 and variance 1
$\hat{R} \equiv \hat{S} \times \hat{T}$	
r	radius from center of planet, kilometers
r_a	radius of apoapsis, kilometers
r_p	radius of periapsis, kilometers
$\hat{S}$	unit vector parallel to incoming hyperbolic asymptote
T	Cartesian covariance matrix of tracking errors
$\hat{T}$	unit vector in planet equator perpendicular to $\hat{S}$
t	time, seconds
t_b	time duration of thrusting maneuver, seconds
U(f,g)	uniform density function defined between f and g
u	random number from uniform distribution defined on interval $[0,1]$
V	integral of acceleration due to thrust
V_a	actual velocity gained, kilometers/second (see eq. (6))
V_b	velocity counting accelerometer bias, meters/second
V_{tbg}	velocity-to-be-gained, kilometers/second (see eq. (5))
V_∞	hyperbolic excess velocity, kilometers/second

v	velocity increment equivalent to one pulse of velocity counting accelerometer, meters/second
v_{cal}	calibrated value of v, meters/second
$\hat{X},\hat{Y},\hat{Z}$	rectangular Cartesian base vectors
$\vec{X}_a$	actual Cartesian state of spacecraft
$\vec{X}_d^R = \vec{X}_n + \Delta\vec{X}_d^R$	
$\vec{X}_e$	estimate of $\vec{X}_a$
$\vec{X}_n$	nominal Cartesian state of spacecraft
x,y,z	rectangular Cartesian coordinates, kilometers
α,β,δ	angles defining initial direction and plane of thrust, degrees (see fig. 1)
γ,ρ,ψ	random variables, degrees (see fig. 6(a))
$\Delta\vec{X}_d \equiv \vec{X}_a - \vec{X}_n$	
$\Delta\vec{X}_e \equiv \vec{X}_e - \vec{X}_n$	
$\Delta\vec{X}_t \equiv \vec{X}_e - \vec{X}_a$	
ϵ	error in the calibration of the velocity counting accelerometer, meters/second
$\vec{\eta}^R$	random vector in principal axis system
Θ	rotation matrix
θ,ϕ	angles of rotation, degrees (see fig. 6(b))
$\dot{\theta}$	thrusting pitch rate, degrees/second (see fig. 1)
λ	eigenvalues
μ	gravitational constant of planet, kilometers^3/second^2

ν	true anomaly, degrees
ν_o	true anomaly at start of maneuver, degrees
ξ	arbitrary variable
Σ	azimuth angle, degrees
σ	standard deviation of random variable (symbols of variables will be used as subscripts to refer to the deviations of specific parameters)
τ	magnitude of thrust, kN
Φ	matrix of eigenvectors
Ω	longitude of ascending node, degrees
ω	argument of periapsis, degrees

Subscripts:

D	D-matrix
T	T-matrix
i,j,k	indices
f	final
n	nominal
o	initial condition
x,y,z	rectangular Cartesian coordinates
1,2,...	first, second, etc.

Superscripts:

j,k	indices

$\rightarrow$	vector
R	random
$\rightarrow$R	random vector
—	mean value
T	matrix transpose
^	unit vector
.	differentiation with respect to time
*	specific value
$', ''\}$	modified parameters

Notation:

~	distributed as
$\|\|\xi\|\|$	denotes integer part of ξ
$P_r[X]$	probability of X occurring
Δ	increment

ANALYSIS

The problem considered in this paper is that of relating the statistics of the errors in the initial orbit and errors in the maneuver to the statistics of the errors in the final orbit where the final orbit results from a minimum fuel, finite burn, orbital transfer. First, the physics of the problem are formulated resulting in a set of three second-order differential equations which represent the equations of motion of the thrusting maneuver. Next, the targeting problem is considered which leads to the numerical values of the six control parameters that characterize the transfer from the initial orbit to the desired final orbit. The statistics of the initial orbit and the maneuver execution errors are then discussed. The Monte Carlo Method of analysis follows which consists of generating a

random sample of the initial orbit, targeting this sample to the desired final orbit, and computing the perturbed final orbit which results from errors in the initial orbit and execution errors. This sequence is repeated many times and the statistics of the errors in the final orbit are formulated.

Equations of Motion

The equations of motion for the transfer trajectory are those of a spacecraft maneuvering with constant thrust τ and mass-flow rate $\dot{m}$ in the neighborhood of a single body. If the mass of the spacecraft at the start of the maneuver is m_o, then the mass at time t is given by

$$m(t) = m_o + \dot{m}t$$

and the magnitude of the acceleration due to thrust by

$$a(t) = \frac{\tau}{m_o + \dot{m}t}$$

The thrust vector is allowed to rotate in a plane with a constant pitch rate $\dot{\theta}$ so that the transfer trajectory is characterized by six control parameters α, β, δ, $\dot{\theta}$, t_b, ν_o (see fig. 1). The orientation of the plane of thrust relative to an inertial $\hat{X},\hat{Y},\hat{Z}$-coordinate

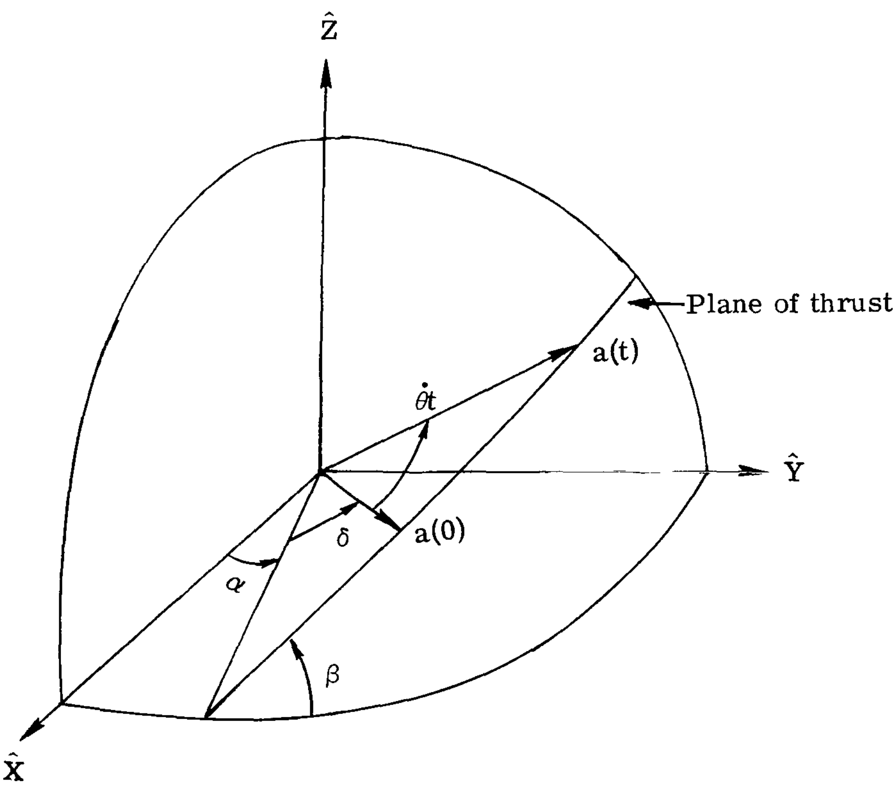


Figure 1.- Sketch of coordinate system and angles used in equations of motion.

system is defined by the two angles α and β. At the start of the maneuver the thrust vector is at an angle δ from the $\hat{X},\hat{Y}$ plane and rotates in the α,β plane at a rate $\dot{\theta}$ until the burn is terminated at $t = t_b$. The direction cosines of the thrust vector at time t are therefore given by

$$\cos(\delta + \dot{\theta}t)\cos\alpha - \sin(\delta + \dot{\theta}t)\sin\alpha\cos\beta$$

$$\cos(\delta + \dot{\theta}t)\sin\alpha + \sin(\delta + \dot{\theta}t)\cos\alpha\cos\beta$$

$$\sin(\delta + \dot{\theta}t)\sin\beta$$

The assumed trajectory model is two-body motion plus an acceleration due to thrust and is defined by the equations of motion

$$\ddot{x} = -\frac{\mu x}{r^3} + a(t)\left[\cos(\delta + \dot{\theta}t)\cos\alpha - \sin(\delta + \dot{\theta}t)\sin\alpha\cos\beta\right]$$

$$\ddot{y} = -\frac{\mu y}{r^3} + a(t)\left[\cos(\delta + \dot{\theta}t)\sin\alpha + \sin(\delta + \dot{\theta}t)\cos\alpha\cos\beta\right]$$

$$\ddot{z} = -\frac{\mu z}{r^3} + a(t)\left[\sin(\delta + \dot{\theta}t)\sin\beta\right]$$

where

$$a(t) = \frac{\tau}{m_o + \dot{m}t}$$

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

The initial conditions for the equations of motion are derived from the knowledge of the initial orbit plus the control parameter ν_o which denotes the true anomaly on the orbit at the start of the maneuver. To conserve computer time the equations of motion are "integrated" by a truncated power series expansion in time (see appendix of ref. 1).

Targeting Problem

The guidance law requires that the control parameters be determined such that the fuel is minimized subject to the requirement that the final orbit satisfies certain constraints. These constraints might consist of requiring the final orbit to have a specific orientation, period, and so forth. Actually, the constraints are chosen from a set of 20 possible target parameters (ref. 1). Since the maneuver is defined by up to six control parameters, at most six constraints can be satisfied. For the fuel to be minimized, the number of constraints must be less than the number of control parameters. The targeting problem, then, is to determine the set of control parameters which satisfy the

constraints imposed on the final orbit and minimize the fuel required for the transfer maneuver. Basically, the targeting involves the solution of a constrained minimization problem by use of constant Lagrange multipliers and the Newton-Raphson iteration technique. The targeter for VEAMCOP is VITAP (ref. 1).

Statistics of the Problem

The statistics of the errors in the initial orbit are defined by two covariance matrices. In the absence of all errors the spacecraft would proceed along the nominal trajectory. However, due to previous perturbations and error sources, the actual path of the spacecraft will not be the nominal trajectory but a neighboring trajectory. This deviation from the nominal trajectory is assumed Gaussian with mean 0 so that the statistics of the actual trajectory can be completely defined by a six-dimensional Cartesian covariance matrix D in position and velocity. For a single off-nominal trajectory, however, the actual state of the spacecraft will not be known. It is assumed that the spacecraft has been tracked and that a filtering process has been used to estimate or predict the actual state of the spacecraft at some time. By the very nature of an estimate it will be in error. Several factors affect the accuracy of the estimate, such as the observability of the spacecraft and the accuracy of the observations. Statistically, the error in the estimate can be defined by a six-dimensional Cartesian covariance matrix T in position and velocity. In order to define the matrices T and D it is first necessary to define the various types of trajectory perturbations. The actual state deviation $\Delta\vec{X}_d$ and the estimate of this deviation $\Delta\vec{X}_e$ are related by (see fig. 2)

$$\Delta\vec{X}_e \equiv \Delta\vec{X}_d + \Delta\vec{X}_t \tag{1}$$

where

$$\Delta\vec{X}_e \equiv \vec{X}_e - \vec{X}_n$$

$$\Delta\vec{X}_d \equiv \vec{X}_a - \vec{X}_n$$

$$\Delta\vec{X}_t \equiv \vec{X}_e - \vec{X}_a$$

Here $\Delta\vec{X}_t$ is the error in obtaining the estimate of the trajectory. With these definitions the matrices D and T can now be defined. As stated earlier D is a six-dimensional covariance matrix describing the statistics of the actual deviations from the nominal or

$$D \equiv E\left[\Delta\vec{X}_d\,\Delta\vec{X}_d{}^T\right] = E\left[\left(\vec{X}_a - \vec{X}_n\right)\left(\vec{X}_a - \vec{X}_n\right)^T\right]$$

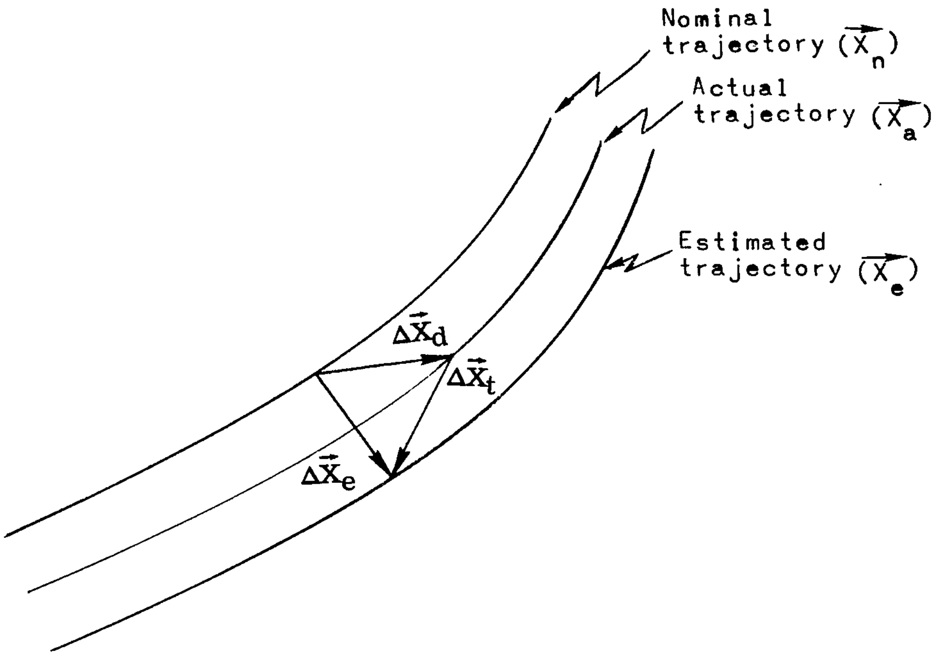


Figure 2.- Sketch of trajectories.

Likewise T is a six-dimensional matrix describing the statistics of the tracking errors, or

$$T \equiv E\left(\Delta\vec{X}_t\ \Delta\vec{X}_t^{\,T}\right) = E\left[\left(\vec{X}_e - \vec{X}_a\right)\left(\vec{X}_e - \vec{X}_a\right)^T\right]$$

Since D is the covariance of $\Delta\vec{X}_d$ and T is the covariance of $\Delta\vec{X}_t$, a sample estimate of the state deviation $\Delta\vec{X}_e$ could be formed by adding a random sample from D to a random sample from T (see eq. (1)). However, this implies that $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$ are uncorrelated which is not the case.[1] It is shown in reference 2 (and used in ref. 3) that for an optimal estimate $E\left(\Delta\vec{X}_e\ \Delta\vec{X}_t^{\,T}\right) = \left[0\right]$ when $\Delta\vec{X}_e \equiv \Delta\vec{X}_d + \Delta\vec{X}_t$. That is, the error in the estimate is uncorrelated with the estimate, thus the covariance between $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$ is

$$\begin{aligned} E\left(\Delta\vec{X}_d\ \Delta\vec{X}_t^{\,T}\right) &= E\left[\left(\Delta\vec{X}_e - \Delta\vec{X}_t\right)\Delta\vec{X}_t^{\,T}\right] \\ &= E\left(\Delta\vec{X}_e\ \Delta\vec{X}_t^{\,T} - \Delta\vec{X}_t\ \Delta\vec{X}_t^{\,T}\right) \\ &= -T \qquad (2) \end{aligned}$$

[1] The assumption that $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$ are uncorrelated is commonly used in Monte Carlo analysis; however, the assumption is incorrect. To the authors' knowledge the correlation between $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$ was first derived for an optimal estimate in 1962 by researchers at Ames Research Center (ref. 3).

An expression for the covariance of $\Delta\vec{X}_e$ can now be found. From equations (1) and (2)

$$E\left(\Delta\vec{X}_e\,\Delta\vec{X}_e^{\,T}\right) = E\left[\left(\Delta\vec{X}_d + \Delta\vec{X}_t\right)\left(\Delta\vec{X}_d + \Delta\vec{X}_t\right)^T\right]$$

$$= E\left(\Delta\vec{X}_d\,\Delta\vec{X}_d^{\,T} + \Delta\vec{X}_t\,\Delta\vec{X}_d^{\,T} + \Delta\vec{X}_d\,\Delta\vec{X}_t^{\,T} + \Delta\vec{X}_t\,\Delta\vec{X}_t^{\,T}\right)$$

$$= D - T \tag{3}$$

Therefore, a random sample of the estimate $\Delta\vec{X}_e^{\,R}$ can be generated from $D - T$. A random error in the estimate $\Delta\vec{X}_t^{\,R}$ can then be generated from T independent of the choice of $\Delta\vec{X}_e^{\,R}$ since $E\left(\Delta\vec{X}_e\,\Delta\vec{X}_t^{\,T}\right) = \left[0\right]$. The random state deviation vector is given by equation (1) as

$$\Delta\vec{X}_d^{\,R} = \Delta\vec{X}_e^{\,R} - \Delta\vec{X}_t^{\,R}$$

During the Monte Carlo process, the random estimate of the initial orbit is targeted to the final orbit which produces a new set of control parameters. These controls are then applied to the actual initial orbit $\left(\vec{X}_a^{\,R} = \vec{X}_n + \Delta\vec{X}_d^{\,R}\right)$ to establish the final orbit. This final orbit, however, will be in error because the control parameters were calculated with the estimate and applied to the actual. In other words, the error in the estimate introduces errors into the final orbit.

Also contributing to the errors in the final orbit are execution errors and the uncertainty in the spacecraft parameters. The control parameters α, β, δ, $\dot{\theta}$, t_b, ν_o cannot be applied or executed exactly as computed because the direction of the thrust will be slightly different than desired, the thrust vector will rotate faster or slower than computed, and so forth. All of these variations will perturb the final orbit. Since the spacecraft will be alined to some arbitrary reference system such as a Sun-Canopus system, an attitude maneuver will be performed to establish the computed direction and plane of thrust which are defined by α, β, and δ. As mentioned, this maneuver will be in error resulting in improper alinement. Not knowing the actual attitude maneuver, it has been assumed that the variation in the direction of thrust is circularly distributed about the computed direction and that the error in the plane of thrust is normally distributed. Thus, the task of relating these error sources to the variation in α, β, and δ arises. This problem is addressed in the appendix. The errors in $\dot{\theta}$ and ν_o have been assumed uncorrelated and normally distributed with mean 0 and variance σ^2, that is

$$\Delta\dot{\theta} \sim N\left(0,\sigma_{\dot{\theta}}^2\right)$$

$$\Delta\nu_o \sim N\left(0,\sigma_{\nu_o}^2\right)$$

The uncertainties in the spacecraft parameters m_o, $\dot{m}$, and τ, and the gravitational constant μ also affect the final orbit and must be considered statistically. These error sources are also assumed uncorrelated and normally distributed, that is

$$\Delta m_o \sim N\left(0,\sigma_{m_o}^2\right)$$

$$\Delta \dot{m} \sim N\left(0,\sigma_{\dot{m}}^2\right)$$

$$\Delta \tau \sim N\left(0,\sigma_{\tau}^2\right)$$

$$\Delta \mu \sim N\left(0,\sigma_{\mu}^2\right)$$

The error model for the total burn time t_b has been assumed representative of a closed loop system; that is, the thrust engine cutoff is triggered by a velocity counting accelerometer. The integrating accelerometer senses equivalent velocity pulses and signals the engine to cut off at the end of K pulses where each pulse represents v meters per second of velocity. There are three sources of error associated with this process. First, the calibrated value of v will be in error by a small amount ϵ where $\epsilon \sim N\left(0,\sigma_\epsilon^2\right)$. Therefore, the true value of $v = v_{cal} + \epsilon$. Secondly, the accelerometer may be biased a small amount V_b where $V_b \sim N\left(0,\sigma_{V_b}^2\right)$. Finally, the accelerometer signals the cutoff on a whole number of counts. Since it goes to the next full count, the velocity added will always be greater than desired but not by more than v_{cal} meters per second. The integral of acceleration due to thrust V is defined as

$$V = \int_0^t a(t)dt = \int_0^t \frac{\tau}{m_o + \dot{m}t}\, dt = \frac{\tau}{\dot{m}} \ln\left(\frac{m_o + \dot{m}t}{m_o}\right) \tag{4}$$

Thus, the velocity-to-be-gained is a function of the computed control parameters and is given by

$$V_{tbg} = \frac{\tau}{\dot{m}} \ln\left(\frac{m_o + \dot{m}t_b}{m_o}\right) \tag{5}$$

The number of pulses is

$$K = \left\|\frac{1000V_{tbg}}{v_{cal}}\right\| + 1$$

where $\|\xi\|$ denotes the integer part of ξ. Now, since the accelerometer was assumed to be slightly biased and since the calibrated value of v is in error, the thrust engine will actually deliver a sensed velocity of

$$V_a = \left[K\left(v_{cal} + \epsilon\right) + V_b\right]\frac{1}{1000} \tag{6}$$

In order to model this effect in the program the change in burn time Δt_b corresponding to V_a is determined from equation (4) as a function of the true values of the engine parameters. These true values are calculated by adding random increments from the respective distributions to the nominal values of τ, m_o, and $\dot{m}$. Equation (4) would then take the form:

$$V_a = \frac{\tau + \Delta\tau}{\dot{m} + \Delta\dot{m}} \ln\left[\frac{\left(m_o + \Delta m_o\right) + \left(\dot{m} + \Delta\dot{m}\right)\left(t_b + \Delta t_b\right)}{m_o + \Delta m_o}\right]$$

where Δt_b is the addition in burn time to compensate for Δm_o, $\Delta\dot{m}$, and $\Delta\tau$. Solving for Δt_b gives

$$\Delta t_b = \left(\frac{m_o + \Delta m_o}{\dot{m} + \Delta\dot{m}}\right)\left[e^{\left(\frac{\dot{m}+\Delta\dot{m}}{\tau+\Delta\tau}\right)V_a} - 1\right] - t_b \tag{7}$$

Monte Carlo Method

In general, the Monte Carlo approach to a statistical problem is to sample repeatedly the statistical distributions of the independent variables, evaluate the function of these variables with the random samples, and accumulate the statistics of the resulting dependent variables. For complicated functions the Monte Carlo method is often the only means by which the statistical nature of the dependent variables can be found.

The Monte Carlo method is very useful in the error analysis of an orbital transfer for the following reasons. The initial orbit is described by two statistical quantities, D and T. In addition, the actual control parameters are known only in the statistical sense. Hence, the final orbit as a function of these various statistical distributions must be described statistically. The problem, then, is to find the statistics of the final orbit and also to determine the statistics of the transfer maneuver and the variation in the fuel resulting from the errors in the initial orbit.

The implementation of the Monte Carlo method to the orbital-transfer error analysis is outlined in figure 3. The first step is to generate a random estimate of the state deviations from the covariance matrix $[D - T]$ and a random error in the estimate from the covariance matrix T. Next, the actual state vector and the estimate of the state vector are formed. During the actual mission, the transfer maneuver will be based on the estimate of the initial orbit since the actual orbit will not be known. To simulate this condition the estimate of the vector state $\vec{X}_e^R$ is targeted to the final orbit with VITAP (ref. 1). This results in a set of up to six control parameters which describe the transfer maneuver.

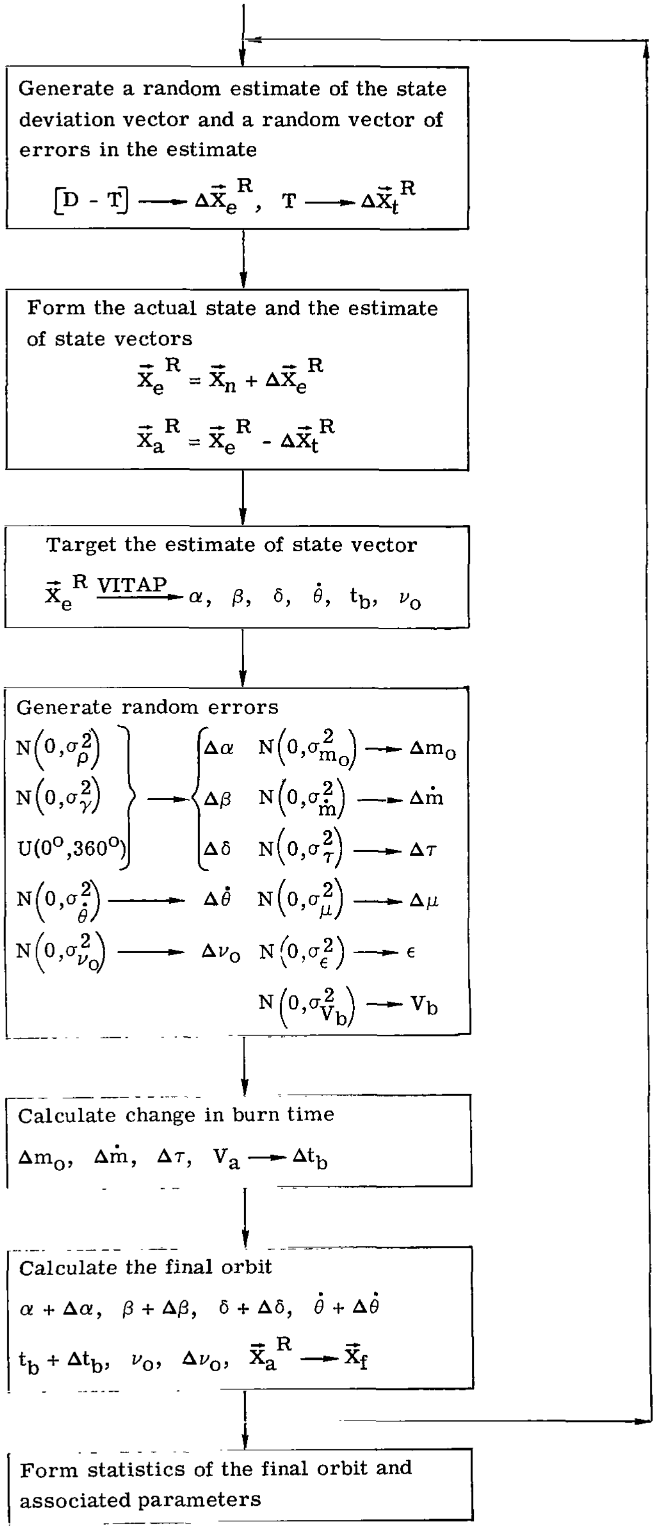


Figure 3.- Flow diagram of the Monte Carlo method as applied to the orbital transfer error analysis.

Since these control parameters cannot be executed exactly, they are perturbed by a random sample from their error distributions. The change in the burn time to correct for errors in m_o, $\dot{m}$, and τ is then calculated from equation (7). These perturbed control parameters are then applied to the sampled, actual initial orbit to establish the final orbit. This entire process is then repeated and another final orbit is established. Finally, after many passes through the Monte Carlo loop, the final orbits are processed and the statistics of the variations in the final orbit are formed.

The accumulation of the statistics of the final orbit and the associated parameters is rather straightforward. If ξ is a dependent variable and the Monte Carlo process has generated N_S samples of ξ denoted by ξ_i $(i = 1, 2, \ldots, N_S)$, then an estimate of the mean of ξ is

$$\bar{\xi} = \frac{1}{N_S} \sum_{i=1}^{i=N_S} \xi_i \tag{8}$$

and an estimate of the variance is

$$\sigma_\xi^2 = \frac{1}{N_S} \sum_{i=1}^{i=N_S} \left(\xi_i - \bar{\xi}\right)^2$$

or

$$\sigma_\xi^2 = \frac{1}{N_S} \sum_{i=1}^{i=N_S} \xi_i^2 - \left(\bar{\xi}\right)^2 \tag{9}$$

If the dependent variable is an M-dimensional vector, say ξ^j, and the N_S samples are denoted by ξ_i^j $(i = 1, 2, \ldots, N_S;\ j = 1, 2, \ldots, M)$ then an estimate of the mean of ξ^j is

$$\overline{\xi^j} = \frac{1}{N_S} \sum_{i=1}^{i=N_S} \xi_i^j \tag{10}$$

and the M by M covariance matrix C_{jk} is

$$C_{jk} = \frac{1}{N_S} \sum_{i=1}^{i=N_S} \left(\xi_i^j - \overline{\xi^j}\right)\left(\xi_i^k - \overline{\xi^k}\right)$$

or

$$C_{jk} = \frac{1}{N_S} \sum_{i=1}^{i=N_S} \xi_i^j \xi_i^k - \overline{\xi^j}\,\overline{\xi^k} \tag{11}$$

The mean and variance of a parameter are very useful quantities to describe the statistical nature of the parameter. A bar chart (see fig. 4) is another way of representing the statistical distribution. Here the range of the parameter is divided into small intervals and the number of samples that fall within each interval is counted. The resulting chart describes the statistical distribution and is called a histogram.

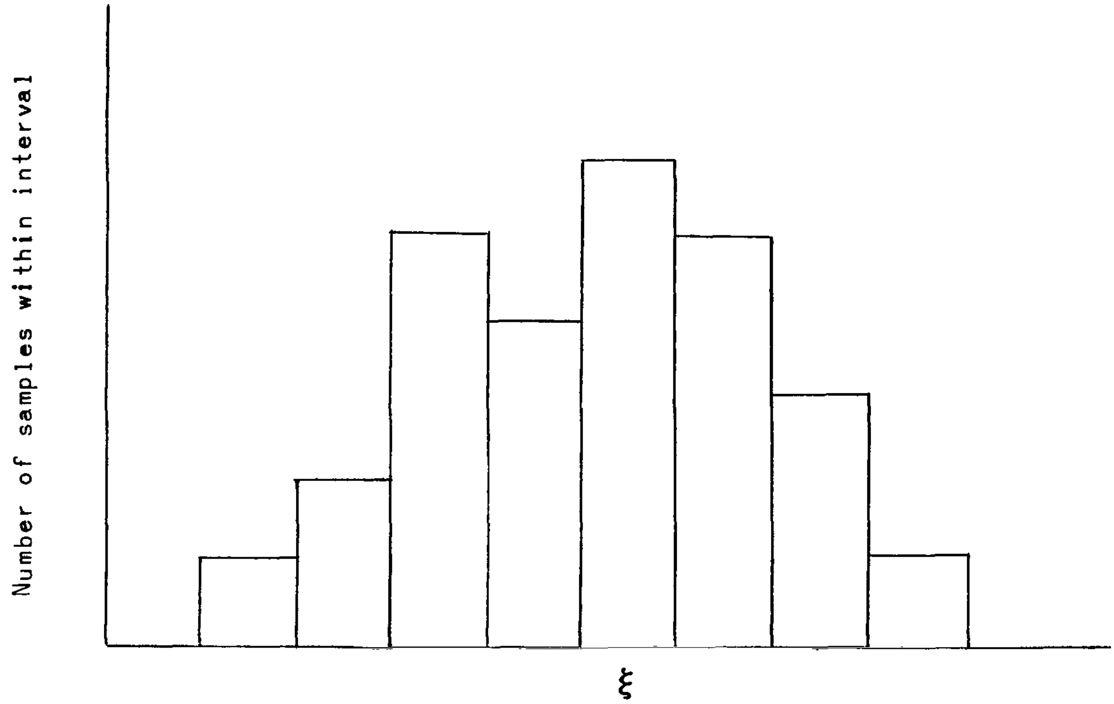


Figure 4.- Bar chart of statistical distribution.

The question still remains concerning the accuracy of the distribution of ξ after N_S samples. For example, how accurate is the estimate for the mean of ξ? (See eq. (8).) It can be shown that as N_S, the number of Monte Carlo samples, increases the difference between the estimate and the true mean decreases. Nevertheless, for a given number of samples it would be desirable to put an interval around $\bar{\xi}$ and be very confident that the true mean was within this interval. If the parametric form of the distribution of ξ were known, the problem would be much easier; however, this is not the case. The problem, then, is to define a confidence interval around the estimate for a nonparametric distribution. These nonparametric confidence intervals of the "central type" are well established (refs. 4 and 5). Briefly, they involve solving the cumulative form of the binomial density for the upper and lower probability limits of the estimate. This probability interval is then mapped into the desired confidence interval about the estimate.

The Monte Carlo process requires the generation of random vectors from the 6 by 6 covariance matrices T and $[D - T]$. This is done in the following way. First

the covariance matrix T is diagonalized by transforming it to the principal axis system, that is

$$\mathrm{diag}\left(\lambda_1, \lambda_2, \ldots, \lambda_6\right) = \Phi^T T \Phi$$

where the λ_i-terms are the eigenvalues of T and the columns of the transformation matrix Φ contain the eigenvectors. The standard deviations in the principal axis system are the square roots of the λ_i-terms so that a random vector $\vec{\eta}^R$ is given by

$$\vec{\eta}^R = \left(R_1\lambda_1^{1/2}, R_2\lambda_2^{1/2}, \ldots, R_6\lambda_6^{1/2}\right)$$

where the R_i-terms are independent random numbers from a normal distribution with mean 0 and variance 1. The random vector $\vec{\eta}^R$ is then rotated back to the original coordinate system to form the desired random vector from T, that is

$$\Delta\vec{X}_t^R = \Phi\vec{\eta}^R$$

A random sample from the covariance matrix $[D - T]$ which is a random sample of the estimate of state deviations $\Delta\vec{X}_e^R$ is found in a similar manner.

The random numbers R_i can be generated in several ways. For this analysis they are found by generating two random numbers u_1 and u_2 from a uniform distribution defined on the interval $[0, 1]$ and evaluating the expression (ref. 6)

$$R_i = \left[-2 \ln u_1\right]^{1/2} \cos(2\pi u_2)$$

Frequently in application the covariance matrices T and $[D - T]$ are found to be indefinite instead of positive definite. This condition usually results from numerical problems in generating the matrices. When the determinant of these matrices is small or when the components of the vector are highly correlated, it is likely that numerical problems have been encountered in computing these matrices and that they are indefinite. If this is the case, then at least one of the eigenvalues is negative and the standard deviation $\sqrt{\lambda_i}$ is meaningless. This problem can be overcome in two ways; the covariance matrix can be forced to be positive definite or the negative eigenvalue can be set equal to 0. The latter was adopted.

COMPUTER PROGRAM VEAMCOP

Program Description

The computational algorithm for a Monte Carlo error analysis of a minimum-fuel, thrusting transfer between two Keplerian orbits has been combined with the computer program VITAP (ref. 1) to form the computer program VEAMCOP (Viking Error Analysis Monte Carlo Program). It contains a main program and seven subroutines in addition to VITAP and its 11 subroutines. The entire program is written in FORTRAN IV

computer language for the Control Data 6600 computer system and resulted in a field length of 73 000_8. A copy of VEAMCOP can be obtained from COSMIC (Computer Software Management and Information Center), Barrow Hall, University of Georgia, Athens, Georgia.

Various options are available in VEAMCOP. The six control parameters α, β, δ, $\dot{\theta}$, t_b, ν_o may be varied in order to minimize the fuel or they may be fixed at specific values. If one or more of these parameters is fixed, then the minimization process varies the other controls to find the best transfer trajectory. The transfer trajectory is also required to satisfy a number of constraints which are outlined in table I and fully discussed in reference 1. The option is available to control such parameters in the final orbit as the six orbital elements, the radius of periapsis and apoapsis, and so forth. In addition, VEAMCOP will perform two modes of targeting. The first mode is the normal minimum-fuel transfer outlined previously. The second mode allows for the minimum-fuel transfer with the additional constraint that the inclination of the final orbit be between an upper and lower bound.

Several input options for the covariance matrices D and T have been incorporated in VEAMCOP and will be discussed with the definition of the program inputs.

Program Input

All input to program VEAMCOP is accomplished by means of a FORTRAN namelist CASE. Each of the namelist variables is defined in table II. Many of the inputs relate to the targeter VITAP and are discussed in reference 1. The remaining parameters are discussed here.

The statistics of the initial orbit T and D are input through the parameters TRACK and DEV, respectively. For convenience three options are available for the input of T and D. The desired option is selected by inputing a 0, 1, or 2 in KEY. KEY equal 0 implies that the input parameter XNOM contains the Cartesian state of the nominal orbit and that the parameters TRACK and DEV contain the full 6 by 6 Cartesian covariance matrices at XNOM. All three parameters, TRACK, DEV, AND XNOM are expressed in the areocentric coordinate system. An input of 1 in KEY implies that XNOM contains the Keplerian orbital elements of the nominal orbit in the areocentric coordinate system. For this option TRACK and DEV contain half of the covariance matrices at XNOM in the $\hat{N},\hat{V},\hat{W}$-coordinate system.[2] Along the diagonals are input the standard

[2] The $\hat{N},\hat{V},\hat{W}$-coordinate system is defined as follows:

$$\hat{N} = \frac{\vec{V}}{|\vec{V}|} \times \frac{\vec{R} \times \vec{V}}{|\vec{R} \times \vec{V}|} \qquad \hat{V} = \frac{\vec{V}}{|\vec{V}|} \qquad \hat{W} = \frac{\vec{R} \times \vec{V}}{|\vec{R} \times \vec{V}|}$$

where $\vec{R}$ is the Cartesian position vector and $\vec{V}$ is the Cartesian velocity vector.

deviations, and the correlation coefficients are defined in the right off-diagonal elements. If this option is exercised, VEAMCOP calculates the full covariance matrices and puts variances along the diagonals and covariances in the off-diagonal elements. It then rotates the covariance matrices from the $\hat{N},\hat{V},\hat{W}$-coordinate system to the areocentric coordinate system. The remaining option, KEY = 2, implies that XNOM contains the Keplerian orbital elements of the nominal orbit in the areocentric coordinate system and that TRACK and DEV contain half-full matrices at XNOME in an arbitrary Mars-centered, inertial, Cartesian coordinate system. The Cartesian state, at which the covariance matrices are defined, XNOME, is expressed in the same coordinate system as TRACK and DEV. Unlike the second option, this option defines the covariance matrices by inputting variances along the diagonals and covariances in the right off-diagonals. When VEAMCOP encounters this option it first fills the left off-diagonal that was not input, rotates the covariance matrices to the $\hat{N},\hat{V},\hat{W}$-coordinate system using the state defined in XNOME, and then transforms the covariance matrices to the areocentric coordinate system by using XNOM. These three options are the result of obtaining the covariance matrices from different computer programs and are included for the convenience of the user.

Most of the remaining inputs are either self-explanatory or well documented in reference 1. The standard deviations of the controls and the spacecraft parameters are input through the array CONT as defined in table II. The input parameter NMC defines the number of Monte Carlo cases or the number of final orbits which will be generated to form the desired statistics. The input quantity MCPRINT is an output option. For each Monte Carlo case a one-line summary of the case is automatically output. In addition, the option is available to output a more extensive summary for each case. If MCPRINT is greater than zero, VEAMCOP will output a block of data for each iteration in the targeting scheme. This output is similar to the VITAP printout and has been included in VEAMCOP as a diagnostic tool. Another automatic feature of VEAMCOP should be mentioned here. The input parameter MAXIT defines the maximum number of iterations allowed for targeting. If the targeter reaches this limit before the process has converged to the minimum fuel controls, then the iteration process is stopped and that particular Monte Carlo case is not considered in the final statistics. However, before proceeding to the next Monte Carlo case, VEAMCOP retargets this divergent case from the beginning and automatically outputs a full description of each iteration. If, however, the MCPRINT option was in effect during the bad case, then the detail output was accomplished during the targeting cycle and there is no need to retarget this case. Finally, the input parameter KOR has been included to control the correlation between $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$. An input of 0 for KOR implies no correlation whereas an input of 1 implies correlation (see eq. (2)).

Many of the input options are more readily defined by their function in VEAMCOP. For this reason a detailed flow diagram of the main program is presented in figure 5.

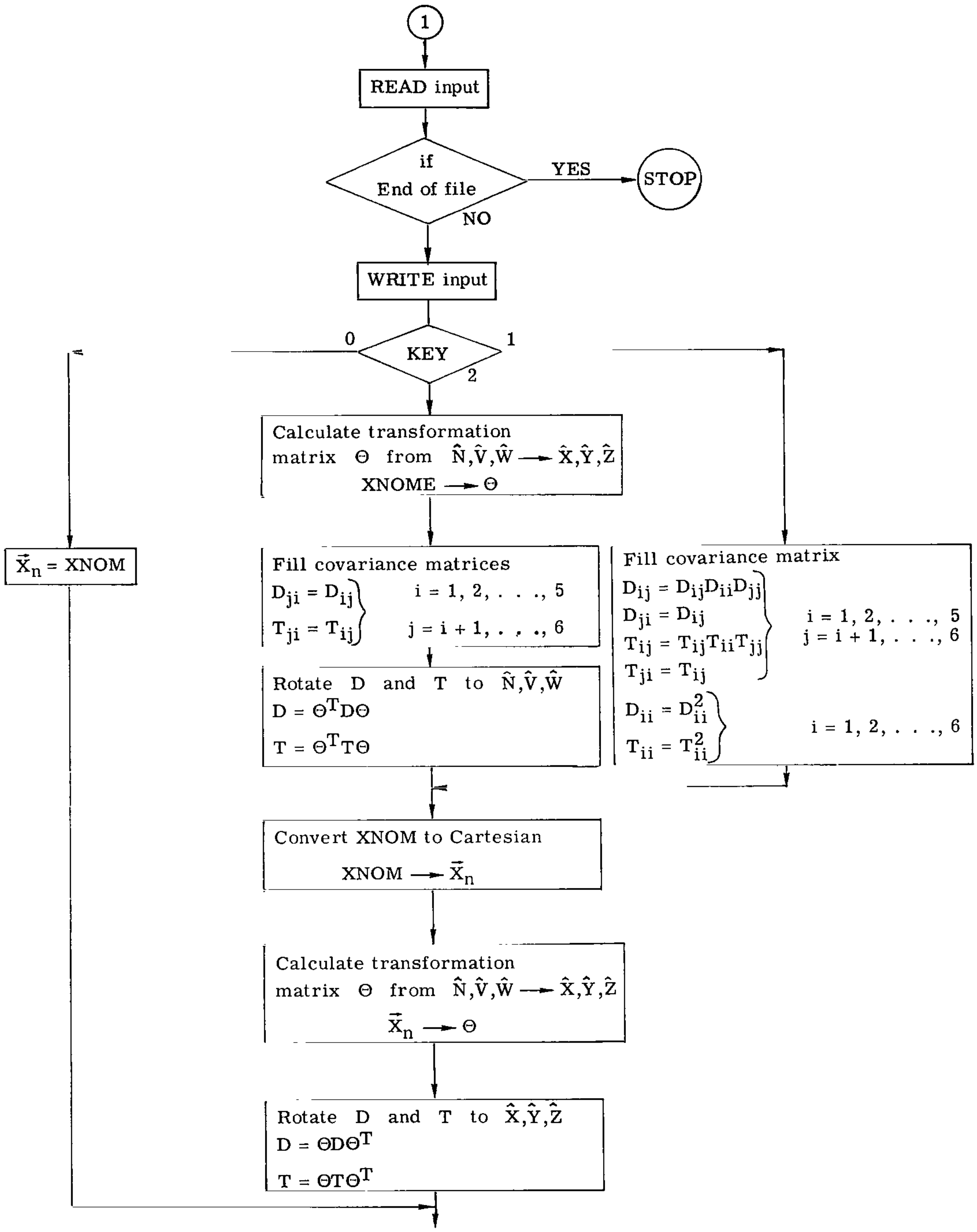


Figure 5.- Flow diagram of main program.

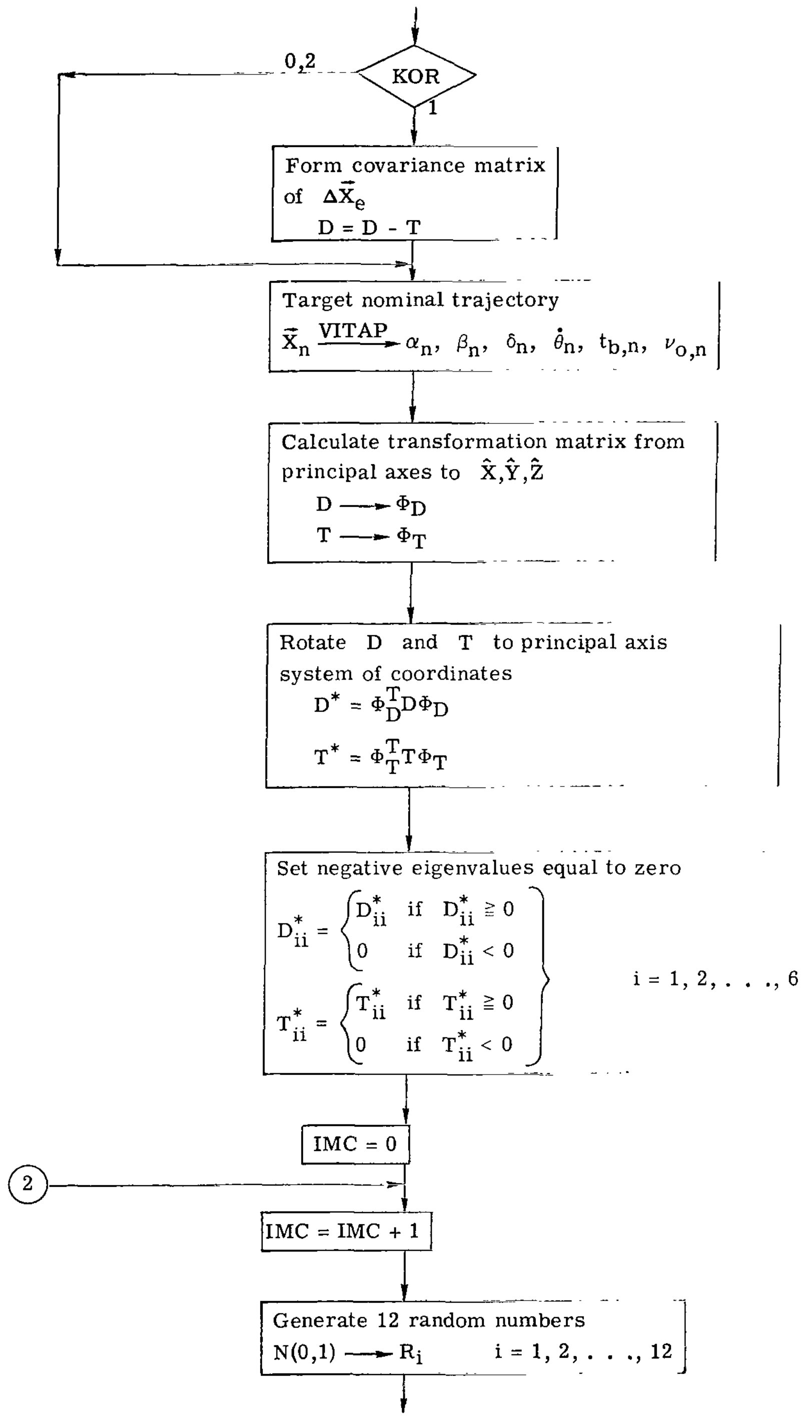


Figure 5.- Continued.

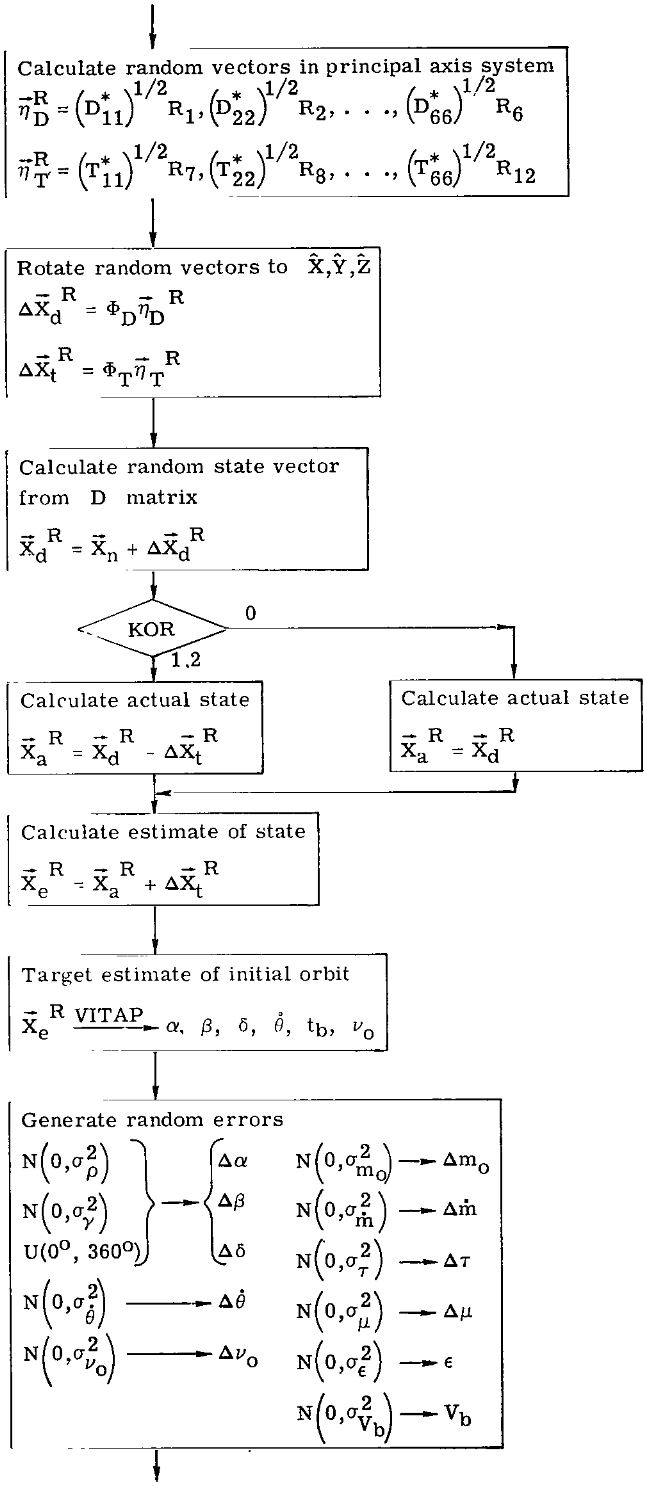


Figure 5.- Continued.

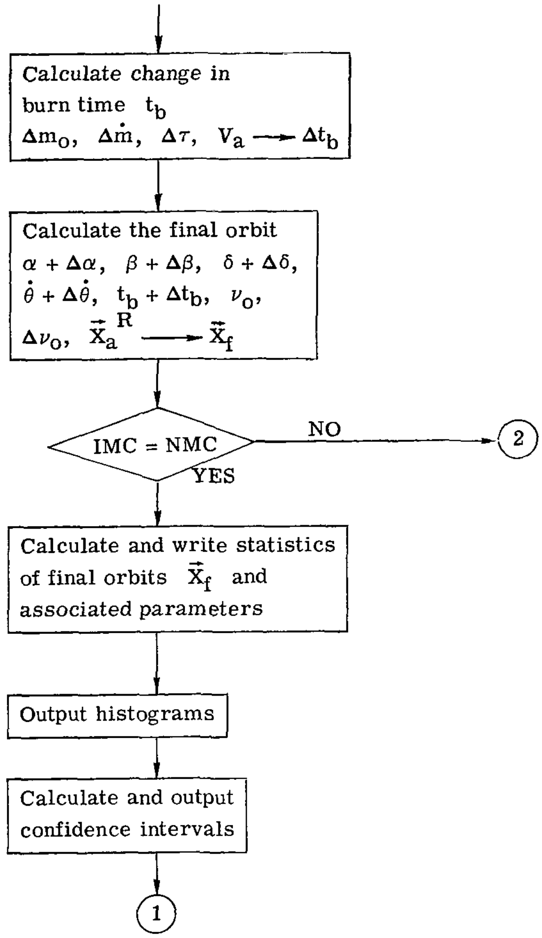


Figure 5.- Concluded.

Program Output and Illustrative Example

A reproduction of a computer output from VEAMCOP is presented in table III. The illustrative example is an error analysis of the Mars orbital insertion maneuver. The spacecraft approaches the planet on a hyperbolic orbit. At an appropriate time, the engine is fired to establish an elliptical orbit. The analysis is concerned with establishing the errors in the ellipse due to variations in the approach hyperbola and in the burn maneuver. In addition, the fuel budget for this ensemble of hyperbolas is determined; that is, how much fuel should be allocated to the insertion maneuver to insure that a large percentage of the off-nominal hyperbolas can be retargeted to the desired ellipse.

The first output from VEAMCOP is a complete listing of all the parameters input through the namelist CASE (table III). For illustrative purposes the covariance matrix of tracking uncertainties, TRACK, and the covariance matrix of state deviations, DEV, are assumed spherical. The tracking uncertainties in position and velocity are 20 km and 20 m/sec (1σ), respectively, and the state deviations in position and velocity are 200 km

and 40 m/sec (1σ), respectively. The value of the input parameter KEY indicates that these matrices are half-full matrices with standard deviations on the diagonals and correlation coefficients in the right off-diagonals. However, since the matrices are spherical, the off-diagonal elements are 0. KEY also indicates that they are expressed in the $\hat{N},\hat{V},\hat{W}$-coordinate system at XNOM which is the array of Keplerian orbital elements of the nominal orbit in the areocentric coordinate system. The sixth entry in XNOM reveals that the covariance matrices are referenced to the point on the hyperbola that is 60° prior to periapsis passage. The input array, CONT, contains the standard deviations of the control parameters, the spacecraft parameters, and the velocity counting accelerometer. The guidance law is defined by the three input arrays: NOPT, KOPT, and AIN. The first six entries in NOPT indicate that four control parameters α, δ, t_b, and ν_0 are free to vary in the minimization scheme to find the optimum transfer maneuver. The remaining two controls, β and $\dot{\theta}$, are held constant at 90° and 0 deg/sec, respectively (see GS array). Therefore, the direction of thrust remains inertially fixed throughout the transfer maneuver at a right ascension of α and a declination of δ (see fig. 1). The next six entries in NOPT indicate that three constraints are imposed on the final orbit. The KOPT array and table I show that the three constraints are $1/a$, i, and ω of the final orbit. These three parameters are constrained to the values in the AIN array. Thus, the guidance law consists of four controls to establish a final orbit with $a = 20\ 455$ km, $i = 35^{\circ}$, and $\omega = 65^{\circ}$. Since the number of constraints is less than the number of controls, the fuel required for the maneuver is minimized. Note from NOPT that the second, fifth, and sixth constraints were not imposed. KOPT and table I indicate that these parameters are the radius of periapsis r_p, the longitude of the ascending node Ω, and the true anomaly ν in the final orbit at the end of the maneuver. Although these target parameters are not constraints, they are nevertheless included as statistical parameters of interest. Also, the namelist printout indicates that 100 Monte Carlo cases were generated (NMC = 100), the first case was output in full (MCPRINT = 1), and that the correlation between $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$ was considered (KOR = 1).

The next section of output is a full description of the targeting of the nominal state, XNOM, to find the nominal controls and the Lagrange multipliers. These nominal values are useful as initial guesses in targeting the off-nominal trajectories. Each iteration of the minimization scheme is recorded by a selected set of output parameters. The control parameters for the first iteration which are the initial guesses input through the GS array are given in the first line of printout followed by the initial guesses at the Lagrange multipliers. These controls correspond to the velocity increment (eq. (4)) labeled "DELTA V." Presented next is the orbit which resulted from applying the initial controls to the nominal orbit. Since the initial guess at the control parameters was not the optimum set, the target parameters are not equal to the desired values. The errors in the target parameters are presented in the next line of output. These errors are then used to calculate

the corrections to the control parameters and the Lagrange multipliers which will improve the initial guesses. The second iteration follows the same format as the first. The new controls are the result of adding the corrections to the previous controls. This process continues until the control parameters converge to the optimum set. For the case considered here, 34 iterations were necessary to target the nominal trajectory.

In the data following the history of the minimization scheme the first line gives the eigenvalues for the second partial sufficiency check (ref. 1). If these values are all greater than 0, then the solution is indeed a local minimum. Negative eigenvalues denote that the solution obtained is not a minimum. For the example case, the minimization process possesses only one degree of freedom since four controls are varied to satisfy three constraints; and, hence, only one eigenvalue exists. Since its value is greater than 0, we are sure that the nominal solution is a minimum solution. This solution is presented on the next line of output followed by the Lagrange multiplier for the nominal case. Finally, the TRACK and DEV covariance matrices are output along with their six-dimensional eigenvalues. If the correlation between DEV and TRACK is considered, then the matrix [DEV-TRACK] is output instead of DEV.

After the nominal state is targeted, the Monte Carlo process begins. From the namelist printout we see that MCPRINT = 1. Therefore, a full description of the first Monte Carlo case is output. The first two lines of output data present the random sample of the actual state and the random estimate of this state. After seven iterations the estimate is retargeted to the desired target parameters. From the second partial check it can be seen that the solution is a minimum. This solution is output next. The actual initial conic is presented next followed by the actual controls. Notice that the "controls due to retargeting the estimate" and the actual controls differ. This is due to the errors associated with the execution of the computed controls. In other words, the actual controls are the result of adding random errors, described by the array CONT, to the computed or retargeted controls. The spacecraft parameters are also perturbed to simulate their uncertainty. The actual thrust and gravitational parameters are presented next. The "actual controls" are then applied to the "actual initial conic" to obtain the "actual target variables" and the "actual final conic." The next two lines of printout are the computed or commanded velocity increment (eq. (5)) and the actual or delivered velocity increment (eq. (6)).

The remaining Monte Carlo cases are summarized with a one-line printout. The first column is the number of the case and the second column is the number of iterations necessary to converge the off-nominal trajectory. The next six columns are the "actual controls" and the next six columns contain the "actual final orbit." The last two columns are the delivered velocity and the number of positive eigenvalues.

The next section of data presents the mean values and the standard deviations of the statistical parameters according to equations (8) to (11). The first row is the expected or mean values of the computed controls followed by the expected values of the actual initial conic. The expected values of the actual final conic and the actual target parameters are presented in the next two rows. The expected values of the commanded and delivered velocity increment are next. Following the expected values are the standard deviations of the same parameters. As a diagnostic aid the reconstructed TRACK and DEV matrices are output. Notice that the mean values of the random vectors generated from these covariance matrices are not 0. This is due to the finite number (NMC) of samples used to calculate these values.

The next section of output is devoted to the histograms of the orbital elements of the final orbit, the six target parameters named in array KOPT, and the orbital elements of the initial orbit. Also included is the histogram of the delivered velocity increment. These histograms give a much better indication of the distribution of the random variables than do the means and standard deviations alone. The plots can be read in the following manner. On the abscissa is plotted the standard deviation of the parameter considered. On the ordinate is plotted the number of Monte Carlo cases that fell between the various standard-deviation intervals. The strange symbols on the graphs can be interpreted in the following manner. If the symbols +--+ appear at the top of a column, the column represents the plotted value on the ordinate. If the symbols +..+ appear, the ordinate is the plotted value plus a number between 0.1 and 0.49 times the ordinate interval value. The symbol ++++ denotes the plotted value plus half of the interval value. The symbol +||+ corresponds to the plotted value plus a number between 0.51 and 0.99 times the ordinate interval value. This interpretation is shown in table IV for interval values of 2, 4, and 8.

After the histograms the cumulative density functions are presented. The first is the cumulative probability of the delivered velocity increment V_a. Here the 100 velocities have been ordered and associated with a probability. The second column is an estimate of the delivered velocity that corresponds to the probability in the first column. The 90-percent confidence interval about this estimate is given in the third and fourth columns. For example, the estimate of V_a that is equal to or greater than 95 percent of the velocities is 1.604 km/sec. This is only an estimate, however, of the true 95th-percentile point. Nevertheless, there is 90-percent confidence that the true 95th-percentile point is between 1.578 km/sec and 1.667 km/sec. To express this mathematically, we define V_a^* as

$$P_r\left[V_a \leqq V_a^*\right] \equiv 0.95$$

and state that

$$P_r\left[1.578 \leqq V_a^* \leqq 1.667\right] = 0.90$$

where the best estimate of V_a is 1.604 km/sec. The final set of data is the cumulative density functions of the six orbital elements of the final orbit and the six target parameters.

CONCLUDING REMARKS

A computer program has been developed which performs an error analysis of a minimum-fuel, finite-thrust, transfer maneuver between two Keplerian orbits. The method of analysis is the Monte Carlo approach where each off-nominal trajectory is targeted to a final conic. Basically, the targeting involves the solution of a constrained minimization problem by use of constant Lagrange multipliers and the Newton-Raphson iteration technique.

The accuracy of the error analysis, within the modeling assumptions, is limited only by the number of Monte Carlo samples generated. In an attempt to model the physical problem, the initial and final orbits have been modeled as Keplerian orbits and the statistics of the initial orbit have been assumed Gaussian. In addition, the errors in the control parameters and physical constants have been modeled as normal random variables. Within these modeling assumptions, the analysis can be performed as accurately as desired at the expense of computer time. As an indication of the accuracy 90-percent confidence intervals are output for some of the statistical quantities.

The program is applicable to the deboost maneuver (hyperbola to ellipse), orbital trim maneuvers (ellipse to ellipse), fly-by maneuvers (hyperbola to hyperbola), escape maneuvers (ellipse to hyperbola), and deorbit maneuvers.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., January 5, 1972.

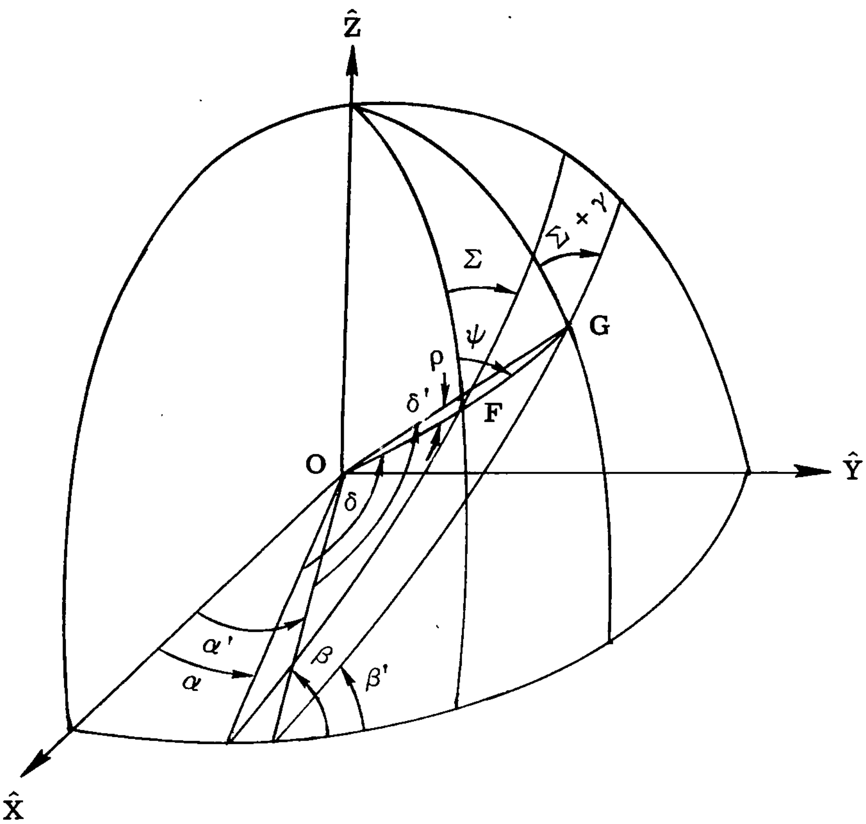
APPENDIX

ERROR MODEL FOR THRUST ANGLES

The initial direction of thrust and the plane of thrust for a maneuvering spacecraft can be defined by three angles α, β, and δ. In order to formulate an accurate error model for these three angles it is necessary to know the history of the maneuvers which alined the spacecraft to the proper attitude. For example, there are four different two-axis maneuvers of pitch, roll, or yaw available to establish the proper direction of thrust. To establish the proper plane of thrust adds additional alternatives. At first it would seem that the maneuver sequence with the smallest sum of rotation angles would minimize the total execution error and thus be the desired sequence. However, the spacecraft attitude control system may be more accurate in one axis than another which would negate this choice. More than likely the maneuver sequence will be required to satisfy certain mission constraints such as antenna nulls, and so forth. Therefore, it is not possible to determine a priori which maneuver sequence will be followed. The only course of action is to assume an error model for the angles α, β, and δ which approximates the true situation.

In this appendix an approximate error model is formulated for both the constant-inertial thrust case and the constant-pitch-rate case. In the constant-inertial thrust case it is assumed that the spacecraft is initially alined to an arbitrary reference coordinate system and that it is rotated through a two-axis maneuver sequence to aline the thrust engine with the line O-F (see fig. 6(a)). It is further assumed that the execution of this maneuver is in error, resulting in the thrust engine being alined with the line O-G. As in the Gates error model (ref. 7) it is assumed that the engine alinement error is circularly distributed about the line O-F. This distribution is modeled by considering ψ as a random variable uniformly distributed between 0° and 360° and by taking ρ as a normal random variable, that is $\rho \sim N\left(0,\sigma_{\rho}^{2}\right)$. The constant-pitch-rate case is similar with the addition of a final roll maneuver to aline the pitch axis perpendicular to the plane of thrust. This roll maneuver is assumed to be in error an amount $\gamma \sim N\left(0,\sigma_{\gamma}^{2}\right)$. Thus, this appendix is concerned with mapping the above error sources into the thrust angles α, β, and δ.

Consider the geometry of figure 6(a). The point F defines the desired initial direction of thrust, and the desired plane of thrust is defined by α and β. Due to execution errors the thrust alinement coincides with point G which is related to point F through the random variables ψ and ρ. The plane of thrust through point G is defined by Σ, the desired azimuth angle, and the random variable γ. It is desired to determine the three angles α', β', and δ' which describe point G.



(a) Points F and G.

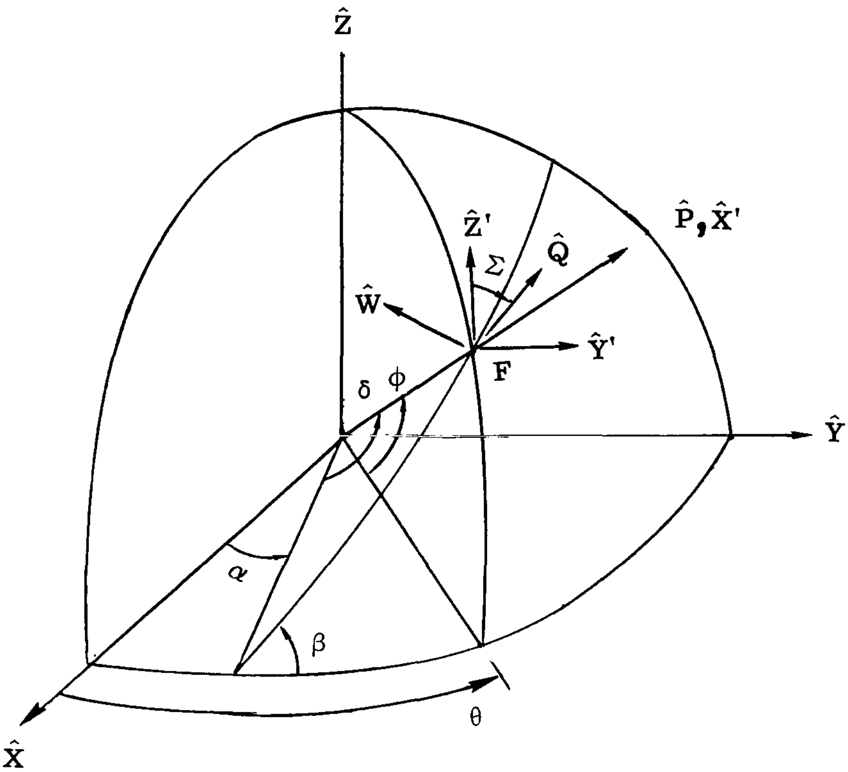
Figure 6.- Geometry of error model.

A useful coordinate system is defined by $\hat{P},\hat{Q},\hat{W}$ (see fig. 6(b)) where $\hat{P}$ is along the radial line to point F, $\hat{Q}$ is perpendicular to $\hat{P}$ and in the α,β plane, and $\hat{W}$ completes the triad. The transformation from the $\hat{P},\hat{Q},\hat{W}$ to the $\hat{X},\hat{Y},\hat{Z}$-coordinate system is given by (ref. 8)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Theta_1 \end{bmatrix} \begin{bmatrix} p \\ q \\ w \end{bmatrix} \tag{A1}$$

where

$$\Theta_1 = \begin{bmatrix} \cos\delta\cos\alpha - \cos\beta\sin\alpha\sin\delta & -\sin\delta\cos\alpha - \cos\beta\sin\alpha\cos\delta & \sin\beta\sin\alpha \\ \cos\delta\sin\alpha + \cos\beta\cos\alpha\sin\delta & -\sin\delta\sin\alpha + \cos\beta\cos\alpha\cos\delta & -\sin\beta\cos\alpha \\ \sin\beta\sin\delta & \sin\beta\cos\delta & \cos\beta \end{bmatrix}$$



(b) Point F.

Figure 6.- Continued.

The unit vector to point F in the $\hat{X},\hat{Y},\hat{Z}$-coordinate system is therefore given by

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \Theta_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

or

$$\left.\begin{aligned} F_x &= \cos\delta\cos\alpha - \cos\beta\sin\alpha\sin\delta \\ F_y &= \cos\delta\sin\alpha + \cos\beta\cos\alpha\sin\delta \\ F_z &= \sin\beta\sin\delta \end{aligned}\right\} \tag{A2}$$

Similarly, the unit vector $\hat{Q}$ is given by

$$\begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix} = \begin{bmatrix} \Theta_1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

or

$$\left.\begin{aligned} Q_x &= -\sin\delta\cos\alpha - \cos\beta\sin\alpha\cos\delta \\ Q_y &= -\sin\delta\sin\alpha + \cos\beta\cos\alpha\cos\delta \\ Q_z &= \sin\beta\cos\delta \end{aligned}\right\} \tag{A3}$$

Now consider the $\hat{X}',\hat{Y}',\hat{Z}'$-coordinate system (see fig. 6(b)) where $\hat{X}'$ is along the radial line to point F, $\hat{Y}'$ is perpendicular to $\hat{X}'$ and in an easterly direction, and $\hat{Z}'$ completes the triad. Note that $\hat{Z}'$ is tangent to the meridian making Σ an azimuth angle. The transformation from the $\hat{X}',\hat{Y}',\hat{Z}'$- to the $\hat{X},\hat{Y},\hat{Z}$-coordinate system can be considered a composite of two rotations. The first rotation is about the $\hat{Z}$-axis an amount θ, and the second rotation is about the new $\hat{Y}$-axis an amount $-\phi$. Thus the desired transformation is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Theta_2 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{A4}$$

where

$$\Theta_2 = \begin{bmatrix} \cos\theta\cos\phi & -\sin\theta & -\cos\theta\sin\phi \\ \sin\theta\cos\phi & \cos\theta & -\sin\theta\sin\phi \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

and

$$\left.\begin{aligned} \sin\theta &= F_y\left(F_x^{\,2} + F_y^{\,2}\right)^{-1/2} \\ \cos\theta &= F_x\left(F_x^{\,2} + F_y^{\,2}\right)^{-1/2} \\ \sin\phi &= F_z \\ \cos\phi &= \left(F_x^{\,2} + F_y^{\,2}\right)^{1/2} \end{aligned}\right\} \tag{A5}$$

If $\hat{F}$ is alined with the $\hat{Z}$-axis, then F_x and F_y are 0 and θ is undefined. In this case θ is set equal to α. The angle Σ (see fig. 6(b)) is the angle from the vector

$\hat{Z}'$ to the vector $\hat{Q}$. The unit vector $\hat{Z}'$ can be expressed in the $\hat{X},\hat{Y},\hat{Z}$-coordinate system as

$$\begin{bmatrix} Z'_x \\ Z'_y \\ Z'_z \end{bmatrix} = \begin{bmatrix} \Theta_2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

or

$$\left.\begin{aligned} Z'_x &= -\cos\theta \sin\phi \\ Z'_y &= -\sin\theta \sin\phi \\ Z'_z &= \cos\phi \end{aligned}\right\} \qquad \text{(A6)}$$

The angle Σ can now be determined by

$$\cos\Sigma = \hat{Q} \cdot \hat{Z}' = Q_x Z'_x + Q_y Z'_y + Q_z Z'_z \qquad \text{(A7a)}$$

$$\sin\Sigma = \left(\hat{Q} \times \hat{Z}'\right) \cdot \hat{F}$$

$$= \left(Q_y Z'_z - Q_z Z'_y\right) F_x + \left(Q_z Z'_x - Q_x Z'_z\right) F_y + \left(Q_x Z'_y - Q_y Z'_x\right) F_z \qquad \text{(A7b)}$$

where $\hat{Q}$, $\hat{Z}'$, and $\hat{F}$ are given by equations (A3), (A6), and (A2), respectively.

It is now desired to determine the coordinates of point G. The angular displacement of G relative to point F is given by the angle ρ, and the azimuth of G is given by ψ. Therefore, the coordinates of $\hat{G}$ in the $\hat{X}',\hat{Y}',\hat{Z}'$-coordinate system are

$$G_{x'} = \cos\rho$$

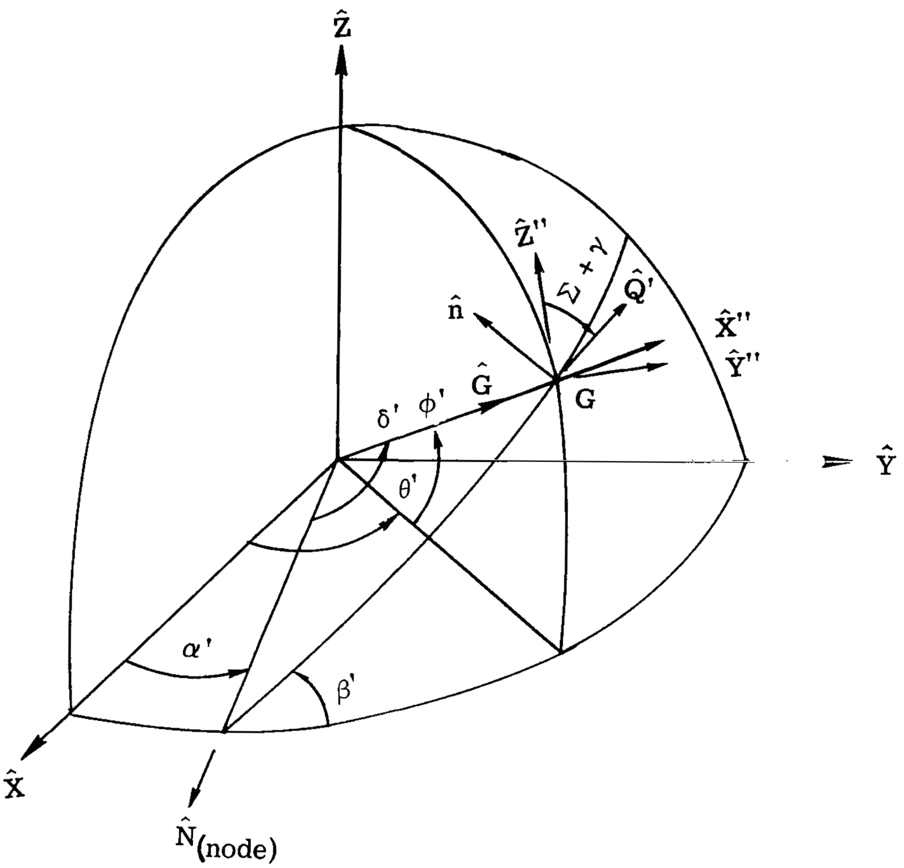
$$G_{y'} = \sin\rho \sin\psi$$

$$G_{z'} = \sin\rho \cos\psi$$

which can be transformed to the $\hat{X},\hat{Y},\hat{Z}$-coordinate system by (eq. (A4))

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \begin{bmatrix} \Theta_2 \end{bmatrix} \begin{bmatrix} G_{x'} \\ G_{y'} \\ G_{z'} \end{bmatrix} \qquad \text{(A8)}$$

Now consider the $\hat{X}'',\hat{Y}'',\hat{Z}''$-coordinate system (see fig. 6(c)) which is defined in the same manner as $\hat{X}'$, $\hat{Y}'$, $\hat{Z}'$. The transformation from the $\hat{X}'',\hat{Y}'',\hat{Z}''$- to the $\hat{X},\hat{Y},\hat{Z}$-coordinate system is given by equations (A4) and (A5) where the components of $\hat{F}$ are



(c) Point G.

Figure 6.- Concluded.

replaced by the components of $\hat{G}$ (eq. (A8)). This transformation is defined by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \Theta_2 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} \tag{A9}$$

Now define $\hat{Q}'$ (see fig. 6(c)) which is the vector that establishes the new plane of thrust. Its components in the $\hat{X}'',\hat{Y}'',\hat{Z}''$-coordinate system are given by

$$Q'_{x''} = 0$$

$$Q'_{y''} = \sin(\Sigma + \gamma)$$

$$Q'_{z''} = \cos(\Sigma + \gamma)$$

which can be transformed by equation (A9)

$$\begin{bmatrix} Q'_x \\ Q'_y \\ Q'_z \end{bmatrix} = \begin{bmatrix} \Theta_2 \end{bmatrix} \begin{bmatrix} 0 \\ \sin(\Sigma + \gamma) \\ \cos(\Sigma + \gamma) \end{bmatrix} \tag{A10}$$

Both the unit vector to the point G (eq. (A8)) and the unit vector in the plane of thrust (eq. (A10)) have been expressed in the $\hat{X},\hat{Y},\hat{Z}$-coordinate system. It remains to determine α', β', and δ'. The vector normal to the plane of thrust $\hat{n}$ is given by $\hat{n} = \hat{G} \times \hat{Q}'$ or

$$n_x = G_y Q'_z - G_z Q'_y$$
$$n_y = G_z Q'_x - G_x Q'_z$$
$$n_z = G_x Q'_y - G_y Q'_x$$

Since the angle between $\hat{n}$ and $\hat{Z}$ is β', we have

$$\cos\beta' = \hat{n} \cdot \hat{Z} = n_z$$

$$\sin\beta' = \sqrt{1 - \cos^2\beta'}$$

where $0^\circ \leqq \beta' \leqq 180^\circ$. The node (see fig. 6(c)) is given by

$$\hat{N} = \frac{\hat{Z} \times \hat{n}}{|\hat{Z} \times \hat{n}|}$$

or

$$N_x = -\frac{n_y}{\sin\beta'}$$

$$N_y = \frac{n_x}{\sin\beta'}$$

$$N_z = 0$$

which yields the angle α', that is

$$\sin\alpha' = N_y$$

$$\cos\alpha' = N_x$$

If $\sin\beta' = 0$, the node is undefined. In this case $\alpha' + \delta' = \theta'$ and it is assumed that $\alpha' = \alpha$ and $\delta' = \theta' - \alpha'$. If this is not the case, the angle δ' is obtained from

$$\cos\delta' = \hat{N} \cdot \hat{G} = N_x G_x + N_y G_y$$

$$\sin\delta' = (\hat{N} \times \hat{G}) \cdot \hat{n} = n_x N_y G_z - n_y N_x G_z + n_z\left(N_x G_y - N_y G_x\right)$$

Thus, given the three desired thrust angles α, β, and δ and the execution errors ψ, ρ, γ, the resulting thrust angles α', β', and δ' can be determined.

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TABLE I.- TARGET PARAMETER REQUEST KEYS

Input value	*Input parameter*					
	KOPT (1)	KOPT (2)	KOPT (3)	KOPT (4)	KOPT (5)	KOPT (6)
1	a	e	i	ω	Ω	ν
2	1/a	r_a	[a] Latitude at reference date	[a] Longitude at reference date	[a] Longitude at reference date	[a] True anomaly at reference date
3	r_a	r_p	$\vec{B} \cdot \hat{T}$	[a] Latitude at reference date	[a] Latitude at reference date	[a] Longitude at reference date
4	Orbital period	$\vec{B} \cdot \hat{R}$		[b] Declination of $\vec{S}$	[b] Right ascension of $\vec{S}$	
5	V_∞			[c] Latitude of landing point	[d] Sun angle at landing point	

[a] The value of PERJD and REFJD must be input (see table II).
[b] $\vec{S}$ ≡ Incoming hyperbolic asymptote.
[c] The value of PER must be input (see table II).
[d] The value of PER, SLAT, and SLON must be input (see table II).

TABLE II.- DEFINITION OF INPUT PARAMETERS FOR PROGRAM VEAMCOP

Program symbol	Mathematical symbol	Dimension	Units	Definition
TRACK	T	(6, 6)	km and sec	Rectangular Cartesian covariance matrix of errors in the estimate of state (see KEY)
DEV	D	(6, 6)	km and sec	Rectangular Cartesian covariance matrix of state deviations (see KEY)
CONT	σ_ρ, σ_γ, blank, $\sigma_{\dot\theta}$ σ_{ν_0}, σ_{m_0}, $\sigma_{\dot m}$, σ_τ σ_μ, σ_ϵ, σ_{V_b}	11	deg, deg, deg/sec, deg, kg, kg/sec, kN, km^3/sec^2, m/sec, m/sec	Standard deviations associated with control parameters and spacecraft parameter; the last two relate to velocity accelerometer (see eq. (6))
XNOM	$\bar{X}_n$	6	deg, km, and sec	Initial nominal orbit expressed in either Cartesian elements or Keplerian orbital elements (see KEY)
NOPT		12	None	Integer array denoting free control variables and selected target parameters, NOPT (1 to 6) corresponds to α, β, δ, $\dot\theta$, t_b, ν_0; NOPT(K) = 0 for Kth control fixed and NOPT(K) = 1 for Kth control free where K = 1, 2, . . ., 6; NOPT (7 to 12) corresponds to constraints; NOPT(K+6) = 0 for Kth constraint omitted; NOPT(K+6) = 1 for Kth constraint considered
KOPT		6	None	Integer array denoting the specific target parameter chosen (see table I)
AIN		6	km, sec, and deg	Array of values for target parameters denoted by KOPT
GS		6	deg and sec	Initial values (guesses) of controls α, β, δ, $\dot\theta$, t_b, ν_0 (see fig. 1); these values will vary or remain fixed depending on NOPT (1 to 6)
GL		6		Initial guesses on Lagrange multipliers; if not input, GL(1 to 6) = 1
HP		6	deg and sec	Increments of controls for numerical partial derivatives; if not input, HP(1 to 6) = 0.6, 0.6, 0.6, 0.006, 1, 0.6
V1		6	rad, sec	Maximum allowable step size for controls during Newton-Raphson iteration; if not input V1(1 to 6) = 0.30, 0.30, 0.30, 0.0003, 50, 0.03
NSTEPS		1	None	Integer denoting number of segments used for power series solution to equation of motion; if not input, NSTEPS = 10
MASS	m_0	1	kg	Initial mass of spacecraft
DMASS	$\dot m$	1	kg/sec	Mass-flow rate
THR	τ	1	kN	Thrust of spacecraft propulsion system
PERJD		1	days	Julian date of periapsis passage on initial conic
REFJD		1	days	Reference Julian date for constraints of longitude, latitude, and true anomaly at a reference time; PERJD and REFJD need not be input if these constraints are not used

TABLE II.- DEFINITION OF INPUT PARAMETERS FOR PROGRAM VEAMCOP – Concluded

Program symbol	Mathematical symbol	Dimension	Units	Definition
PER		1	deg	Angular distance from periapsis to landing point
SLAT		1	deg	Declination of subsolar point in areocentric equatorial coordinate system
SLON		1	deg	Right ascension of subsolar point; input SLAT and SLON only if KOPT(5) = 5
ERR		1	None	Newton-Raphson convergence criteria; if not input, ERR = 10^{-6}
MAXIT		1	None	Integer denoting maximum number of iterations allowed; if not input, MAXIT = 50
UMARS	μ	1	km^3/sec^2	Mars gravitational constant; if not input, UMARS = 42 828.4
MODE		1	None	Denotes program mode: 1 – normal forward targeting; 2 – forward targeting, inclination within bounds; 3 – not allowed in VEAMCOP
BOUND		2	deg	Bounds on inclination for MODE = 2; BOUND(1) is lower bound
NMC	N_s	1	None	Integer number of Monte Carlo cases considered
KEY		1	None	Integer denoting input options for TRACK, DEV, XNOM; KEY = 0 implies XNOM is the Cartesian state of nominal orbit and DEV and TRACK are full covariance matrices at XNOM; XNOM, DEV, and TRACK are in areocentric coordinate system; KEY = 1 implies XNOM is Keplerian orbital elements of nominal orbit in areocentric and DEV and TRACK are half-full matrices (standard deviations on diagonal, correlation coefficients in right off diagonal) in N,V,W-coordinate system at XNOM; KEY = 2 implies XNOM is Keplerian orbital elements of nominal orbit in areocentric and DEV and TRACK are half-full matrices (variances on diagonal, covariances in right off diagonal) in any Cartesian system at XNOME; DEV and TRACK may be full matrices
MCPRINT		1	None	Number of Monte Carlo cases for which iterations are output; MCPRINT $\geqq 0$
KOR		1	None	Integer denoting correlation; KOR = 0 implies no correlation between $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$, that is $E\left(\Delta\vec{X}_e \, \Delta\vec{X}_e^T\right) = D + T$; KOR = 1 implies correlation between $\Delta\vec{X}_d$ and $\Delta\vec{X}_t$, that is $E\left(\Delta\vec{X}_e \, \Delta\vec{X}_e^T\right) = D - T$; KOR = 2 used only on case following KOR = 1 and implies correlation; this option implies that the desired covariance matrices have been established in the first case and used for the second case; DEV and TRACK are not input when KOR = 2
XNOME		6	km and sec	Cartesian state of XNOM in same coordinate as DEV and TRACK; XNOME input only when KEY = 2
VCAL	v_{cal}	1	m/sec	Calibrated value of velocity counting accelerometer

TABLE III.- SAMPLE OUTPUT

```
$CASE

TRACK   =  0.2E+02,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.2E+02,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.2E+02,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.2E-01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.2E-01,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.2E-01,

DEV     =  0.2E+03,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.2E+03,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.2E+03,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,
           0.4E-01,  0.0,  0.0,  0.0,  0.0,  0.0,  0.0,  0.4E-01,  0.0,  0.0,
           0.0,  0.0,  0.0,  0.0,  0.4E-01,

CONT    =  0.5E+00,  0.0,  0.0,  0.0,  0.2E+00,  0.32069E+02,  0.477E-02,
           0.1E-02,  0.14E+01,  0.1E-03,  0.28E+00,

XNOM    = -0.426E+04,  0.211E+01,  0.3689E+02,  0.6377E+02,  0.5195E+02,
          -0.6E+02,

NOPT    =  1,  0,  1,  0,  1,  1,  1,  0,  1,  1,  0,  0,

KOPT    =  2,  3,  1,  1,  1,  1,

AIN     =  0.48887802493 27E-04,  0.0,  0.35E+02,  0.65E+02,  0.0,  0.0,

GS      =  0.28E+02,  0.9E+02,  -0.15E+02,  0.0,  0.26E+04,  -0.7E+02,

GL      =  0.1E+01,  0.0,  0.1E+01,  0.1E+01,  0.0,  0.0,

HP      =  0.1E-01,  0.1E-01,  0.1E-01,  0.1E-04,  0.1E+01,  0.1E-01,

V1      =  0.3E+00,  0.3E+00,  0.3E+00,  0.3E-03,  0.5E+02,  0.3E+00,

NSTEPS  =  6,

MASS    =  0.320695E+04,

DMASS   = -0.4772E+00,

THR     =  0.13345E+01,

PERJD   = I,

REFJD   =  0.0,

PER     = I,

SLAT    = I,

SLON    = I,

ERR     =  0.1E-09,

MAXIT   =  100,

UMARS   =  0.428284E+05,

MODE    =  1,

BOUND   = I, I,

NMC     =  100,

KEY     =  1,

MCPRINT =  1,

KOR     =  1,

XNOME   = I, I, I, I, I, I,

VCAL    =  0.3E-01,

$END
```

TABLE III.- SAMPLE OUTPUT – Continued

```
TARGETING NOMINAL STATE TO FIND NOMINAL CONTROLS AND MULTIPLIERS

INITIAL CONIC
 SMA -4260.0000      ECC  2.1100000      INC  36.890000      PER  63.770000      NOD  51.950000      TAN  300.00000

ITERATION  1-------------------------------------------- DELTA V     1.368064761
CONTROLS    23.0000000000           90.0000000000          -15.0000000000      0.                   2600.0000000         -70.0000000000
LGR MULT    1.0000000000            0.                     1.0000000000        1.00000000000        0.                   0.
ORBIT       24692.1256068           .807302626007          37.2517975971       67.2646287248        52.5702261015        34.3537807809
ERRORS      -9.38906148[illegible]E-06  0.                 2.25179759714       2.26462872478        0.                   0.
CORRECT     -3.6        0.          -4.3        0.         -50         15      -4.46E+05  0.        1.6         .21      0.          0.

ITERATION  2-------------------------------------------- DELTA V     1.334333511
CONTROLS    18.7743108862           90.0000000000          -19.3324629253      0.                   2550.0000000         -54.8974861284
LGR MULT    -446467.213802          0.                     2.56132875187       1.21494833450        0.                   0.
ORBIT       57542.1218294           .915149894540          37.0064503720       69.5468356004        53.2995414614        57.2522071066
ERRORS      -3.151526707904E-05     0.                     2.00645037198       4.54683560043        0.                   0.
CORRECT     2.4         0.          -1.2        0.         -50         -1.4    -5.23E+04  0.        1.3         5.51E-02 0.          0.

ITERATION  3-------------------------------------------- DELTA V     1.301004279
CONTROLS    20.801993218[illegible] 90.0000000000          -20.5251504628      0.                   2500.0000000         -56.2486963898
LGR MULT    -438724.59395[illegible] 0.                    3.89289191317       1.27003103563        0.                   0.
ORBIT       57577.3[illegible]42527 .915993838663          36.9259226106       69.5111303843        52.2260537405        54.9287831792
ERRORS      -3.15198480[illegible]E-05  0.                 1.92592261056       4.51113038426        0.                   0.
CORRECT     1.7         0.          -.15        0.         -50         -2.8    -8.58E+04  0.        -1.2        6.64E-02 0.          0.

ITERATION  4-------------------------------------------- DELTA V     1.268067594
CONTROLS    22.[illegible]          90.0000000000          -20.6758969302      0.                   2450.0000000         -59.0150191514
LGR MULT    -364309.377[illegible]  0.                     2.68452182872       1.33644321952        0.                   0.
ORBIT       5757[illegible]         .916880582714          36.8547457400       69.5255791404        51.7872424336        50.0323209872
ERRORS      -3.15[illegible]E-05    0.                     1.85474573998       4.52557914042        0.                   0.
CORRECT     1.5         0.          .67         0.         -50         -3.8    -2.52E+05  0.        -1.5        -3.54E-02 0.         0.

ITERATION  5-------------------------------------------- DELTA V     1.235514317
CONTROLS    24.[illegible]          90.0000000000          -20.0047940586      0.                   2400.0000000         -62.8548882749
LGR MULT    -336[illegible]         0.                     1.18514930169       1.30100527593        0.                   0.
ORBIT       5[illegible]118.0737073 .919963622117          36.8049510976       69.6152513203        51.7134306248        41.8697212269
ERRORS      -3.[illegible]E-05      0.                     1.80495109760       4.61525132026        0.                   0.
CORRECT     [illegible] 0.          1.8         0.         -50         -6.3    -1.15E+06  0.        -1.1        -.24     0.          0.

ITERATION  6-------------------------------------------- DELTA V     1.203335625
CONTROLS    27.[illegible]          90.0000000000          -18.2275318168      0.                   2350.0000000         -69.1322447153
LGR MULT    -[illegible]            0.                     7.49559205846E-02   1.05902196731        0.                   0.
ORBIT       [illegible]             .937440193700          36.7621157004       69.8547372703        51.7691630779        24.3896349320
ERRORS      -3.[illegible]          0.                     1.76211570038       4.85473727031        0.                   0.
CORRECT     -5.5        0.          -4.8        0.         -50         13      -1.41E+07  0.        1.6         -2.4     0.          0.

ITERATION  7-------------------------------------------- DELTA V     1.171522995
CONTROLS    [illegible]             90.0000000000          -22.9774488250      0.                   2300.0000000         -56.0717190758
LGR MULT    -[illegible]            0.                     1.67625485014       -1.32672928359       0.                   0.
ORBIT       -213[illegible]         1.00[illegible]39913   36.7753465942       70.6682048696        51.3944846745        48.3994895493
ERRORS      -5.[illegible]E-05      0.                     1.77534659419       5.66820486965        0.                   0.
CORRECT     1.2         0.          .94         0.         -50         -4.3    -3.26E+06  0.        9.8         2.1      0.          0.
```

TABLE III.- SAMPLE OUTPUT – Continued

ITERATION 8	----------	----------	DELTA V	1.140068192		
CONTROLS	[illegible]	[illegible]	-22.0354229991	0.	2250.00000000	-60.3695316234
LGR MULT	[illegible]	0.	11.4599331144	.779273739795	0.	0.
ORBIT	[illegible]	[illegible]	36.7452919074	70.7201569799	51.5334526009	39.9234679204
ERRORS	[illegible]	0.	1.74525190736	5.72015597985	0.	0.
CORRECT	[illegible] 0.	1.6 0.	-50 -5.8	-1.39E+07 0.	-1.3 -2.0	0. 0.
ITERATION 9	----------	----------	DELTA V	1.108963255		
CONTROLS	[illegible]	[illegible]	-20.3990999641	0.	2200.00000000	-66.1856608568
LGR MULT	[illegible]	0.	10.2009837061	-1.19416312377	0.	0.
ORBIT	[illegible]	1.0138773240	36.7121449153	70.8429593905	51.6700836790	24.9481617939
ERRORS	[illegible]	0.	1.71214481527	5.84295939045	0.	0.
CORRECT	[illegible] 0.	6.2 0.	7.9 -17	-1.79E+09 0.	-1.33E+02 -6.72E+02	0. 0.
ITERATION 10	----------	----------	DELTA V	1.113770422		
CONTROLS	[illegible]	[illegible]	-14.2397801808	0.	2207.76371777	-83.3743947107
LGR MULT	[illegible]	0.	-123.086604509	-672.925594387	0.	0.
ORBIT	[illegible]	1.00732162127	36.5985767575	67.1796571653	51.9589686461	321.278861313
ERRORS	[illegible]	0.	1.59857635751	2.17965715527	0.	0.
CORRECT	[illegible] 0.	.12 0.	-50 3.7	-1.13E+03 0.	-35 1.19E+02	0. 0.
ITERATION 11	----------	----------	DELTA V	1.082955019		
CONTROLS	[illegible]	[illegible]	-14.1155539826	0.	2157.76371777	-80.6713325747
LGR MULT	[illegible]	0.	-157.996312915	-553.590478535	0.	0.
ORBIT	[illegible]	1.19957779207	36.5594242869	66.9859839977	51.9054893918	331.794005995
ERRORS	[illegible]	0.	1.55942425686	1.98598399771	0.	0.
CORRECT	[illegible] 0.	-.12 0.	-50 2.9	-2.04E+08 0.	65 2.00E+02	0. 0.
ITERATION 12	----------	----------	DELTA V	1.052475480		
CONTROLS	[illegible]	[illegible]	-14.2206565895	0.	2107.76371777	-77.7357228413
LGR MULT	[illegible]	0.	-92.7649038604	-353.332185344	0.	0.
ORBIT	[illegible]	1.1304426956	36.5181647759	66.7387457790	51.8305616528	342.728239762
ERRORS	[illegible]	0.	1.51816477585	1.73874577898	0.	0.
CORRECT	1.77E-02 0.	-.43 0.	-50 3.4	-3.57E+08 0.	2.93E+02 3.07E+02	0. 0.
ITERATION 13	----------	----------	DELTA V	1.022324561		
CONTROLS	[illegible]	[illegible]	-14.6602401759	0.	2057.76371777	-74.3432606294
LGR MULT	[illegible]	0.	200.353160790	-47.1301479344	0.	0.
ORBIT	[illegible]	1.1571037124	36.4733543319	65.5831813365	51.7201630909	354.423300554
ERRORS	[illegible]	0.	1.47335431392	1.58318133651	0.	0.
CORRECT	-.03 0.	-.96 0.	-50 4.4	-7.15E+08 0.	9.37E+02 4.88E+02	0. 0.
ITERATION 14	----------	----------	DELTA V	.9924952522		
CONTROLS	[illegible]	[illegible]	-15.6158649130	0.	2007.76371777	-69.9171503818
LGR MULT	[illegible]	0.	1137.62935341	441.190471897	0.	0.
ORBIT	[illegible]	1.1553077308	36.4214460447	65.3536481301	51.5320284449	7.95539858658
ERRORS	[illegible]	0.	1.42144604465	1.35364813008	0.	0.
CORRECT	-1.4 0.	-1.9 0.	-50 9.1	-2.55E+09 0.	5.01E+03 1.17E+03	0. 0.
ITERATION 15	----------	----------	DELTA V	.9629307650		
CONTROLS	[illegible]	[illegible]	-19.4661610160	0.	1957.76371777	-60.7976884238
LGR MULT	[illegible]	0.	6148.41940233	1515.37047052	0.	0.
ORBIT	[illegible]	1.1372508134	36.3610581514	66.0415621715	50.9510362875	29.9965271838
ERRORS	[illegible]	0.	1.36105815136	1.04156217155	0.	0.
CORRECT	[illegible] 0.	-1.0 0.	50 1.5	-3.24E+09 0.	1.25E+04 5.72E+02	0. 0.

TABLE III.- SAMPLE OUTPUT – Continued

```
ITERATION 16-------------------------------------------- DELTA V       .9924952522
CONTROLS    33.1079179402       90.0000000000       -19.4692438466      0.                  2007.76371777       -59.3065330289
LGR MULT    -2105254544.24      0.                  17442.3453238       2187.25733597       0.                  0.
ORBIT       -42313.1227731      1.11072413973       36.2738632956       65.9710326334       50.5458584494       35.4734357203
ERRORS      -7.2330580298351E-05 0.                 1.27386329564       .971032633369       0.                  0.
CORRECT     .76       0.        -.35       0.       50        -.27      -2.93E+09   0.      1.51E+04  1.05E+03  0.        0.

ITERATION 17-------------------------------------------- DELTA V       1.022324561
CONTROLS    33.7692583007       90.0000000000       -19.8313898919      0.                  2057.76371777       -59.5766089522
LGR MULT    -1222776063.5       0.                  32771.6611803       3241.8159714        0.                  0.
ORBIT       -54775.3414674      1.06672405149       36.1847237774       65.8978090545       50.2623506264       37.3729737867
ERRORS      -6.7160607217275E-05 0.                 1.18472377742       .837809054548       0.                  0.
CORRECT     .75       0.        -.21       0.       50        -.65      -2.88E+09   0.      1.79E+04  1.57E+03  0.        0.

ITERATION 18-------------------------------------------- DELTA V       1.052475480
CONTROLS    34.6201133497       90.0000000000       -20.0419736438      0.                  2107.76371777       -60.2279063669
LGR MULT    -1494376497.7       0.                  50650.5190653       4815.07265630       0.                  0.
ORBIT       -73264.7465085      1.06318543506       36.0945816973       65.8251258681       50.0144849178       38.5075228011
ERRORS      -5.2177705629106E-05 0.                 1.09458169726       .825125868107       0.                  0.
CORRECT     .59       0.        -.14       0.       50        -.80      -2.84E+09   0.      1.97E+04  2.00E+03  0.        0.

ITERATION 19-------------------------------------------- DELTA V       1.082955019
CONTROLS    35.3146721382       90.0000000000       -20.1778211173      0.                  2157.76371777       -61.0325132209
LGR MULT    -1774235769 8.5     0.                  70305.5960831       6817.98642107       0.                  0.
ORBIT       -120779.444201      1.03958230983       36.0042787674       65.7534915542       49.7900738988       39.2946274360
ERRORS      -5.7215632518215E-05 0.                 1.00427876737       .753491554214       0.                  0.
CORRECT     .62       0.        -9.36E-02  0.       50        -.87      -2.81E+09   0.      2.07E+04  2.33E+03  0.        0.

ITERATION 20-------------------------------------------- DELTA V       1.113770422
CONTROLS    35.9475615829       90.0000000000       -20.2703992575      0.                  2207.76371777       -61.9011269808
LGR MULT    -2054830393.8       0.                  90957.1595274       9146.38367519       0.                  0.
ORBIT       -294872.175011      1.01613255629       35.9141740754       65.6829592045       49.5832495871       39.8924101790
ERRORS      -5.2279102469145E-05 0.                 .914174075407       .682959204529       0.                  0.
CORRECT     .55       0.        -6.60E-02  0.       50        -.89      -2.78E+09   0.      2.11E+04  2.56E+03  0.        0.

ITERATION 21-------------------------------------------- DELTA V       1.144929173
CONTROLS    36.4945089550       90.0000000000       -20.3364009613      0.                  2257.76371777       -62.7907459550
LGR MULT    -2332362839.2       0.                  112026.761224       11701.7177647       0.                  0.
ORBIT       653877.065787       .992634728111       35.8244067211       65.6134656891       49.3900745743       40.3776294311
ERRORS      -4.7351414052878E-05 0.                 .824406721148       .613465689084       0.                  0.
CORRECT     .49       0.        -4.93E-02  0.       50        -.89      -2.75E+09   0.      2.11E+04  2.70E+03  0.        0.

ITERATION 22-------------------------------------------- DELTA V       1.176439009
CONTROLS    36.9832044943       90.0000000000       -20.3857205031      0.                  2307.76371777       -63.6788765732
LGR MULT    -2607098494 3.6     0.                  133099.312976       14397.4998081       0.                  0.
ORBIT       154741.037214       .969200601834       35.7350196194       65.5449248913       49.2076444201       40.7921998720
ERRORS      -4.242539266326E-05 0.                  .735019619394       .544924891276       0.                  0.
CORRECT     .43       0.        -3.88E-02  0.       50        -.87      -2.72E+09   0.      2.08E+04  2.76E+03  0.        0.

ITERATION 23-------------------------------------------- DELTA V       1.208307932
CONTROLS    37.4123562437       90.0000000000       -20.4245647070      0.                  2357.76371777       -64.5533571893
LGR MULT    -2879263766 7.5     0.                  153880.098460       17160.0590592       0.                  0.
ORBIT       87777.1800099       .945648889115       35.6460186410       65.4772582968       49.0337530736       41.1606415506
ERRORS      -3.7495319847 02E-05 0.                 .646018641010       .477258296755       0.                  0.
CORRECT     .38       0.        -3.24E-02  0.       50        -.85      -2.70E+09   0.      2.03E+04  2.77E+03  0.        0.
```

TABLE III.- SAMPLE OUTPUT – Continued

```
ITERATION 24---------------------------------------------- DELTA V    1.240544221
CONTROLS   37.7875563859      90.0000000000     -20.4569612279    0.                 2407.76371777     -65.4075913255
LGR MULT   -31490730300.8     0.                174159.549427      19927.2691447     0.                0.
ORBIT      51231.9733727      .922003273454     35.5574024482      65.4104056169     48.8666992510     41.4980220169
ERRORS     -3.2516460183978E-05 0.              .557402448228      .410405616360     0.                0.
CORRECT    .33      0.        -2.86E-02  0.      50       -.83      -2.68E+09  0.      1.94E+04  2.72E+03  0.      0.

ITERATION 25---------------------------------------------- DELTA V    1.273156445
CONTROLS   38.1179176557      90.0000000000     -20.4855806965    0.                 2457.76371777     -66.2380968438
LGR MULT   -34167707657.2     0.                193787.115855      22646.7261799     0.                0.
ORBIT      46995.3274217      .358241131089     35.4691795947      65.3443293785     48.7051558760     41.8140001523
ERRORS     -2.7504780069335E-05 0.              .469179584723      .344329378489     0.                0.
CORRECT    .28      0.        -2.66E-02  0.      50       -.81      -2.66E+09  0.      1.89E+04  2.63E+03  0.      0.

ITERATION 26---------------------------------------------- DELTA V    1.306153475
CONTROLS   38.3958411706      90.0000000000     -20.5122157952    0.                 2507.76371777     -67.0431633253
LGR MULT   -36823783912.5     0.                212652.134732      25273.8738856     0.                0.
ORBIT      38093.7573674      .874342543172     35.2813817769      65.2790192932     48.5480806518     42.1150567269
ERRORS     -2.2605781553130E-05 0.              .381381776857      .279019293169     0.                0.
CORRECT    .24      0.        -2.59E-02  0.      50       -.78      -2.64E+09  0.      1.80E+04  2.50E+03  0.      0.

ITERATION 27---------------------------------------------- DELTA V    1.339544501
CONTROLS   38.6370695506      90.0000000000     -20.5380725364    0.                 2557.76371777     -67.8220668661
LGR MULT   -39419930333.4     0.                230669.180483      27770.1366865     0.                0.
ORBIT      32071.5359330      .950239339213     35.2940804684      65.2145005569     48.3946581000     42.4058371046
ERRORS     -1.7648210269705E-05 0.              .294080488393      .214500556875     0.                0.
CORRECT    .20      0.        -2.79E-02  0.      50       -.75      -2.62E+09  0.      1.71E+04  2.33E+03  0.      0.

ITERATION 28---------------------------------------------- DELTA V    1.373339045
CONTROLS   38.8465753015      90.0000000000     -20.5639526130    0.                 2607.76371777     -68.5745503177
LGR MULT   -42063151033.2     0.                247764.668865      30100.7463196     0.                0.
ORBIT      27567.0000937      .826653858744     35.2074178170      65.1508532300     48.2442706605     42.6901077250
ERRORS     -1.2603557764270E-05 0.              .207417817045      .150853199952     0.                0.
CORRECT    .17      0.        -2.64E-02  0.      50       -.73      -2.60E+09  0.      1.61E+04  2.13E+03  0.      0.

ITERATION 29---------------------------------------------- DELTA V    1.407546980
CONTROLS   39.1042281816      90.0000000000     -20.5903687458    0.                 2657.76371777     -69.3003360194
LGR MULT   -44567823457.2     0.                263858.768259      32231.1472988     0.                0.
ORBIT      24077.3808196      .801545557650     35.1216842242      65.0882646869     48.0965130822     42.9717922562
ERRORS     -7.6037373279936E-06 0.              .121684224210      8.3264686869486E-02 0.              0.
CORRECT    .13      0.        -2.72E-02  0.      50       -.70      -2.59E+09  0.      1.50E+04  1.89E+03  0.      0.

ITERATION 30---------------------------------------------- DELTA V    1.442178544
CONTROLS   39.3462330775      90.0000000000     -20.6176162530    0.                 2707.76371777     -69.9981035375
LGR MULT   -47274993583.8     0.                278817.352058      34116.3766238     0.                0.
ORBIT      21572.4729642      .777008223461     35.0376011184      65.0272255783     47.9513419782     43.2574541195
ERRORS     -2.5384457476195E-05 0.              3.7601115368000E-02 2.7225578330226E-02 0.             0.
CORRECT    4.17E-03  0.       -1.39E-02  0.      25       -.32      -1.27E+09  0.      6.53E+03  7.37E+02  0.      0.

ITERATION 31---------------------------------------------- DELTA V    1.459430668
CONTROLS   39.1574773352      90.0000000000     -20.6315481658    0.                 2732.44170106     -70.3234344897
LGR MULT   -48547598648.5     0.                285249.322725      34853.3679762     0.                0.
ORBIT      20415.3477761      .764589910495     34.9953879949      64.9996015045     47.8821518837     43.4221822080
ERRORS     -8.3117660031017E-11 0.              -6.1200505206225E-04 -3.9849545645588E-04 0.           0.
CORRECT    -7.51E-03  0.      8.21E-05  0.       -2.71E-02  2.23E-03  2.57E+05  0.     -1.08E+02  -24     0.      0.
```

TABLE III.- SAMPLE OUTPUT – Continued

```
ITERATION 32------------------------------------------------ DELTA V        1.459411656
CONTROLS      39.1789673012       90.0000000000       -20.6314660638       0.                  2732.41458892      -70.3212006043
LGR MULT     -4854042977?.4       0.                  285240.844036        34829.3657651       0.                 0.
ORBIT         20415400159?4       .764587799944        34.9999999599        64.9999996455       47.9826256659      43.4265164775
ERRORS       -3.5?8147?7?3149?-13 0.                 -4.9079060259177E-08 -3.54473741026cE-07 0.                 0.
CORRECT      -7.35E-07  0.        5.66E-09  0.         2.90E-05 -3.43E-07 -1.57E+03  0.       -3.41E-02 -3.19E-03  0.          0.

ITERATION 33------------------------------------------------ DELTA V        1.459411676
CONTROLS      39.1789593653       90.0000000000       -20.6314660552       0.                  2732.41461792      -70.3212009472
LGR MULT     -485424342?3?5.?     0.                  285240.809914        34829.3625735       0.                 0.
ORBIT         20454.3400998       .764537581644        35.0000000000        65.0000000001       47.9826257868      43.4265163467
ERRORS        4.3845137875?3??-15 0.                  1.8189894035465E-12  1.127773430198E-10  0.                 0.
CORRECT       1.34E-10  0.        3.78E-11  0.        -3.08E-09  5.71E-11  -2.5       0.        1.95E-05  3.81E-06  0.          0.

ITERATION 34------------------------------------------------ DELTA V        1.459411676
CONTROLS      39.1789593653       90.0000000000       -20.6314660551       0.                  2732.41461792      -70.3212009472
LGR MULT     -48542424838.4       0.                  285240.809933        34829.3625773       0.                 0.
ORBIT         20455.0000000       .764637581645        35.0000000000        65.0000000000       47.9826257868      43.4265163468
ERRORS        3.67?6177?76884E-13 0.                  0.                  -2.7284841053196E-12 0.                 0.
CORRECT       1.77E-12  0.       -1.31E-11  0.        -2.51E-11  1.59E-11   3.3       0.       -1.72E-05 -2.38E-06  0.          0.

EIGENVALUES FOR SECOND PARTIAL CHECK  3.0117??85E+06

NOMINAL CONTROLS
 ALPHA  39.178960       BETA  90.000000       DELTA -20.631466       THDOT 0.          TBURN  2732.4146        TAN -70.321201

NOMINAL MULTIPLIERS
  -4854?424835.?     0.                   285240.809916       34829.3625749       0.                   0.

TRACK MATRIX
   400.00000000      -2.1574874429?2E-13  3.8543890885825E-13  0.                    0.                    0.
  -5.794844570491?E-13  400.00000000     -5.5951140328688E-13  0.                    0.                    0.
   7.2591938?3541?E-13 -1.0378034156?3E-12  400.00000000        0.                    0.                    0.
   0.                 0.                  0.                    4.000000000000E-04   -4.727193493724E-19    1.896159836599E-19
   0.                 0.                  0.                   -2.706590324535E-19    4.000000000000E-04   -1.353783768620E-18
   0.                 0.                  0.                    1.082352494640E-19   -9.464676390061E-19    4.000000000000E-04

DEV-TRACK MATRIX
   39600.000000      -5.4265770337?9E-11  5.9504408177?4E-11   0.                    0.                    0.
  -5.8245076703653E-11  39600.000000     -1.4744333960625E-10  0.                    0.                    0.
   8.0277862233440E-11 -1.30663839433E-10  39600.000000        0.                    0.                    0.
   0.                 0.                  0.                    1.200000000000E-03   -1.418158048117E-18    5.688479509798E-19
   0.                 0.                  0.                   -8.119770973605E-19    1.200000000000E-03   -4.061351305859E-18
   0.                 0.                  0.                    3.247087483921E-19   -2.839402917018E-18    1.200000000000E-03

EIGENVALUES OF TRACK
   400.00000000       400.00000000        400.00000000          4.000000000000E-04    4.000000000000E-04    4.000000000000E-04

EIGENVALUES OF DEV-TRACK
   39600.000000       39600.000000        39600.000000          1.200000000000E-03    1.200000000000E-03    1.200000000000E-03
```

TABLE III.- SAMPLE OUTPUT – Continued

```
CASE      1

ACTUAL
   X  4015.7041          Y  5769.3798          Z  365.11756        XD -4.1950086        YD -1.0727570         ZD  2.0021508

ESTIMATE
   X  4035.3120          Y  5745.1756          Z  360.97606        XD -4.2056749        YD -1.0434784         ZD  2.0037876

ITERATION  1------------------------------------------ DELTA V        1.459411676
CONTROLS     39.1789595655        90.0000000000        -20.6314660551        0.                       2732.41461792          -70.3212009472
LGR MULT   -48540424805.1       0.                     285240.809916        34829.3625749      0.                      0.
ORBIT        24943.7369857        .809554550116         35.6608850208       65.0666501319       46.9175979162           46.6059983810
ERRORS     -8.797578619772E-06  0.                      .660885020795       6.665013192378E-02  0.                      0.
CORRECT      .89        0.        -.14         0.          50          -1.5        1.92E+08  0.          2.18E+04  6.59E+03  0.          0.

ITERATION  2------------------------------------------ DELTA V        1.494695623
CONTROLS     40.0737493568        90.0000000000        -20.7686045966        0.                       2782.41461792          -71.7815389208
LGR MULT   -48348178646.1       0.                     307009.879594        41424.2823889      0.                      0.
ORBIT        22777.5018599        .790597210245         35.3374523055       65.0394608468       46.6403191395           44.7919811832
ERRORS     -4.934831651682E-06  0.                      .337452305463       3.946084678228E-02  0.                      0.
CORRECT      .67        0.        -.13         0.          50          -1.2        4.32E+07  0.          1.24E+04  4.51E+03  0.          0.

ITERATION  3------------------------------------------ DELTA V        1.530430444
CONTROLS     40.7446108284        90.0000000000        -20.9024424225        0.                       2832.41461792          -72.9845598329
LGR MULT   -48304937090.7       0.                     319364.455769        45936.4334420      0.                      0.
ORBIT        20905.6139178        .770946802157         35.0452923291       65.0089130048       46.3968321153           43.4036153632
ERRORS     -1.053761171549E-06  0.                    4.529232912864E-02   8.913004802707E-03  0.                      0.
CORRECT    8.19E-02   0.        -3.11E-02      0.          12          -.19       -3.46E+07  0.          4.98E+02  3.61E+02  0.          0.

ITERATION  4------------------------------------------ DELTA V        1.533779294
CONTROLS     40.8263975737        90.0000000000        -20.9335090436        0.                       2844.00450430          -73.1779724210
LGR MULT   -48339569191.1       0.                     319862.257150        46297.6191810      0.                      0.
ORBIT        20455.8093768        .765757210649         34.9995217237       65.0000593599       46.3453746879           43.3097770240
ERRORS     -1.934348023409E-09  0.                   -4.782763408002E-04   5.935988974670E-05  0.                      0.
CORRECT    -1.25E-03  0.         6.56E-05      0.       -9.98E-03   1.52E-03   -3.83E+04  0.          -24       -11        0.          0.

ITERATION  5------------------------------------------ DELTA V        1.538772097
CONTROLS     40.8251511924        90.0000000000        -20.9334434176        0.                       2843.99452803          -73.1764505166
LGR MULT   -48339607506.7       0.                     319838.543064        46286.5875866      0.                      0.
ORBIT        20455.0013256        .765759597682         34.9999999917       64.9999394687       46.3454869805           43.3140621955
ERRORS     -3.168219159244E-12  0.                   -8.299139153678E-09  -5.312967914506E-07  0.                      0.
CORRECT    -6.66E-07  0.        -1.96E-07      0.        2.21E-05  -2.72E-07   -2.18E+03  0.         -1.23E-02 -4.40E-03  0.          0.

ITERATION  6------------------------------------------ DELTA V        1.538772113
CONTROLS     40.8251505268        90.0000000000        -20.9334436134        0.                       2843.99455017          -73.1764507883
LGR MULT   -48339609686.3       0.                     319838.530771        46286.5831875      0.                      0.
ORBIT        20454.9999998        .765759582583         35.0000000000       65.0000000002       46.3454870263           43.3140618135
ERRORS      5.152128723651E-15  0.                   -1.818989403546E-12   1.555235940032E-10  0.                      0.
CORRECT     1.59E-10  0.         6.18E-11      0.       -3.22E-09   6.15E-11   -4.8       0.          3.30E-05  6.04E-06  0.          0.

ITERATION  7------------------------------------------ DELTA V        1.538772113
CONTROLS     40.8251505269        90.0000000000        -20.9334436133        0.                       2843.99455016          -73.1764507882
LGR MULT   -48339609691.0       0.                     319838.530804        46286.5831935      0.                      0.
ORBIT        20455.0000000        .765759582586         35.0000000000       65.0000000000       46.3454870263           43.3140618137
ERRORS      1.303042606983E-13  0.                    0.                  -9.094947017729E-13  0.                      0.
CORRECT     8.39E-12  0.        -1.59E-11      0.       -1.85E-11   1.79E-11    2.0       0.         -1.53E-05 -3.45E-06  0.          0.
```

TABLE III.- SAMPLE OUTPUT – Continued

```
EIGENVALUES FOR SECOND PARTIAL CHECK  5.38820071E+06

CONTROLS DUE TO RETARGETING ESTIMATE
 ALPHA  40.825151       BETA  90.000000      DELTA -20.933444      THDOT 0.              TBURN  2843.9946       TAN -73.176451

ACTUAL INITIAL CONIC
 SMA -4044.8768      ECC  2.1533582      INC  37.571445      PER  64.470445      NOD  51.289223      TAN  300.40966

ACTUAL CONTROLS
 ALPHA  40.817443       BETA  90.000000      DELTA -20.891884      THDOT 0.              TBURN  2795.5240       TAN -73.500759

ACTUAL THRUST AND GRAVITATIONAL PARAMETERS
MASS  3146.8506    DMASS -.47685832      THR  1.3331260       MU  42829.704

ACTUAL TARGET VARIABLES
 4.99233448982 8E-05   4791.78453643      35.0111103246      66.0397161349      46.7689647178      39.4867025182

ACTUAL FINAL CONIC
 SMA  20030.709      ECC  .76077809      INC  35.011110      PER  66.039716      NOD  46.768965      TAN  39.486703

COMMANDED DELTA V   1.53877211276

DELIVERED DELTA V   1.54036650642
```

TABLE III.- SAMPLE OUTPUT – Continued

SUMMARY OF REMAINING CASES

NMC	NIT	ALPHA	BETA	DELTA	THDOT	TBURN	TA	SMA	ECC	INC	ARGP	NODE	TA	DV	NPE
2	7	32.56	90.00	-16.78	0.00000	2596.1	-68.77	20249.1	.77209	34.81	66.28	52.22	44.84	1.378	1
3	7	37.06	90.00	-17.46	0.00000	2645.8	-71.20	23463.9	.81106	34.64	64.42	50.81	51.68	1.393	1
4	8	50.81	90.00	-19.20	0.00000	2808.7	-75.96	24572.0	.80454	35.46	63.39	42.05	40.94	1.500	1
5	6	37.46	90.00	-19.72	0.00000	2705.2	-71.69	18459.7	.75136	35.33	65.58	51.42	45.23	1.449	1
6	8	44.25	90.00	-21.87	0.00000	2896.4	-73.19	20923.3	.75811	35.46	63.45	44.27	38.24	1.576	1
7	7	27.38	90.00	-19.49	0.00000	2807.4	-70.41	19117.8	.74280	35.18	65.59	55.16	34.38	1.534	1
8	7	24.01	90.00	-15.93	0.00000	2749.8	-70.05	20771.6	.76213	34.90	64.92	57.11	30.26	1.449	1
9	7	27.73	90.00	-15.83	0.00000	2710.2	-73.62	18705.5	.75791	34.66	67.65	56.04	36.19	1.463	1
10	7	32.91	90.00	-20.19	0.00000	2907.8	-72.07	18573.1	.74143	34.96	66.99	51.05	42.85	1.548	1
11	8	23.78	90.00	-16.75	0.00000	2704.1	-70.96	20256.7	.75924	35.41	65.56	58.96	25.27	1.443	1
12	7	30.09	90.00	-16.00	0.00000	2780.6	-69.71	19964.1	.75987	35.24	63.38	52.69	44.96	1.474	1
13	8	23.09	90.00	-13.90	0.00000	2708.6	-73.37	20325.4	.77296	35.18	64.47	59.13	31.01	1.434	1
14	7	41.72	90.00	-21.62	0.00000	2843.6	-73.19	20958.5	.76818	34.88	65.43	45.15	41.25	1.565	1
15	6	37.12	90.00	-17.18	0.00000	2680.6	-72.71	20281.7	.78283	35.77	66.33	48.94	50.47	1.422	1
16	9	50.71	90.00	-20.83	0.00000	2892.5	-79.69	20131.5	.76558	34.32	65.33	43.31	35.67	1.587	1
17	6	32.53	90.00	-20.37	0.00000	2726.4	-69.07	22049.5	.77738	34.67	63.66	50.87	40.40	1.474	1
18	6	34.42	90.00	-21.46	0.00000	2852.5	-65.64	26466.9	.79678	35.04	61.80	47.81	41.37	1.521	1
19	7	31.53	90.00	-19.37	0.00000	2857.3	-69.13	21879.8	.77082	34.85	64.77	51.33	41.75	1.541	1
20	7	43.22	90.00	-19.49	0.00000	2623.4	-70.93	20205.6	.77034	34.88	64.47	48.15	44.86	1.379	1
21	7	49.25	90.00	-21.57	0.00000	2838.7	-77.25	20947.0	.77279	35.18	65.67	43.39	37.17	1.547	1
22	6	33.90	90.00	-19.31	0.00000	2675.7	-67.59	20867.1	.76649	34.55	65.21	48.30	43.61	1.429	1
23	8	46.91	90.00	-20.34	0.00000	2877.0	-76.32	22620.1	.78661	34.77	64.09	43.96	39.35	1.589	1
24	6	38.57	90.00	-18.66	0.00000	2686.3	-70.31	25177.7	.81703	34.99	62.77	50.50	49.98	1.423	1
25	9	35.70	90.00	-16.01	0.00000	2518.3	-69.45	19482.1	.77470	34.47	65.40	52.47	49.67	1.320	1
26	5	38.65	90.00	-21.37	0.00000	2778.2	-71.13	21076.5	.77072	34.69	64.63	46.77	43.30	1.505	1
27	7	48.54	90.00	-20.14	0.00000	2732.9	-76.28	20634.7	.77314	34.70	65.51	44.90	37.76	1.489	1
28	9	53.57	90.00	-17.83	0.00000	2913.0	-82.01	20458.2	.77992	34.93	64.78	41.29	42.51	1.573	1
29	11	53.03	90.00	-18.22	0.00000	3083.0	-80.64	21929.7	.77567	34.96	64.95	39.42	37.86	1.718	1
30	7	32.01	90.00	-20.01	0.00000	2590.8	-65.98	19598.2	.74414	34.81	66.10	50.34	35.96	1.409	1
31	5	44.44	90.00	-19.99	0.00000	2710.8	-72.19	21481.6	.77711	34.75	65.22	45.40	41.63	1.449	1
32	6	36.30	90.00	-20.68	0.00000	2789.0	-69.60	19975.7	.75202	34.46	64.76	48.52	41.76	1.522	1
33	7	28.21	90.00	-17.59	0.00000	2685.7	-70.56	20609.5	.76680	34.54	64.55	54.86	34.50	1.458	1
34	6	42.41	90.00	-20.40	0.00000	2673.2	-72.07	21710.6	.78400	34.48	65.09	46.36	42.71	1.413	1
35	6	44.24	90.00	-19.22	0.00000	2813.6	-74.89	21410.0	.78425	34.60	64.69	45.71	46.35	1.513	1
36	7	47.13	90.00	-18.62	0.00000	2846.8	-75.95	19862.7	.76936	35.14	64.49	45.48	47.66	1.507	1
37	5	42.38	90.00	-20.53	0.00000	2711.3	-71.66	21780.4	.78287	35.41	65.56	47.59	43.30	1.437	1
38	6	43.75	90.00	-20.53	0.00000	2730.0	-71.78	18814.7	.74177	35.33	64.88	44.81	40.32	1.440	1
39	6	43.37	90.00	-20.60	0.00000	2666.5	-71.59	20392.4	.76877	34.99	64.91	45.74	43.03	1.432	1
40	6	36.30	90.00	-18.73	0.00000	2735.7	-68.99	22149.7	.78342	35.56	64.11	48.78	47.37	1.452	1
41	7	28.65	90.00	-18.80	0.00000	2695.3	-67.77	19389.5	.74308	35.66	64.68	55.06	35.25	1.440	1
42	7	28.15	90.00	-18.01	0.00000	2702.9	-68.67	21841.7	.77400	34.50	65.20	54.76	35.35	1.449	1
43	8	27.63	90.00	-14.06	0.00000	2644.7	-70.50	20973.9	.77614	35.01	66.02	56.44	36.40	1.409	1
44	6	30.84	90.00	-22.13	0.00000	2737.1	-66.95	20246.7	.74730	34.92	63.77	52.19	36.41	1.463	1
45	8	41.01	90.00	-21.56	0.00000	2992.0	-72.13	18184.7	.71925	35.07	65.53	43.76	41.02	1.604	1
46	7	36.37	90.00	-18.96	0.00000	2592.2	-65.96	21557.3	.77384	35.42	64.68	49.94	43.75	1.368	1
47	6	35.03	90.00	-20.04	0.00000	2785.6	-68.88	18813.4	.73649	35.32	65.63	50.41	41.44	1.498	1
48	8	43.51	90.00	-19.97	0.00000	2919.1	-75.89	21578.3	.76761	34.61	64.69	42.05	36.67	1.598	1
49	6	39.26	90.00	-20.23	0.00000	2654.2	-67.63	21697.4	.77383	35.12	65.98	48.20	42.45	1.403	1
50	6	42.41	90.00	-20.91	0.00000	2775.7	-69.91	17959.5	.72252	35.29	65.99	44.77	41.36	1.480	1
51	7	36.55	90.00	-19.24	0.00000	2866.0	-70.61	19465.6	.75005	35.11	66.39	48.33	46.05	1.554	1
52	8	43.37	90.00	-19.47	0.00000	2811.2	-77.45	18141.8	.74656	34.68	65.11	43.82	42.46	1.516	1

TABLE III.- SAMPLE OUTPUT – Continued

53	6	36.10	90.00	-19.83	0.00000	2627.1	-68.69	21692.5	.78233	34.60	64.66	49.48	45.05	1.404	1
54	8	24.36	90.00	-15.82	0.00000	2651.3	-71.23	19477.0	.75781	35.73	64.20	57.63	31.46	1.397	1
55	5	49.70	90.00	-19.97	0.00000	2776.2	-70.19	19643.3	.75235	35.62	65.09	45.44	45.01	1.458	1
56	10	48.47	90.00	-19.91	0.00000	3038.5	-79.65	20282.2	.76875	35.62	64.20	41.33	43.83	1.671	1
57	10	45.38	90.00	-20.51	0.00000	2997.5	-75.66	18020.9	.71630	34.53	65.66	40.68	35.29	1.667	1
58	8	39.63	90.00	-19.55	0.00000	2550.7	-71.27	19125.0	.77341	35.64	66.49	49.97	50.13	1.317	1
59	6	39.79	90.00	-19.67	0.00000	2587.1	-70.33	20903.0	.78149	34.98	66.13	49.91	45.54	1.402	1
60	8	30.06	90.00	-16.25	0.00000	2545.6	-69.89	20443.4	.77363	35.51	64.15	55.38	37.72	1.370	1
61	7	38.76	90.00	-23.44	0.00000	2896.2	-67.80	24996.0	.78402	35.35	64.33	44.97	38.02	1.558	1
62	8	37.57	90.00	-20.97	0.00000	2514.3	-68.75	19633.6	.76664	35.23	67.56	50.24	42.56	1.326	1
63	8	42.09	90.00	-20.47	0.00000	2562.7	-72.62	22767.4	.80945	34.76	63.55	50.14	49.25	1.338	1
64	7	39.82	90.00	-20.98	0.00000	2861.1	-72.63	19050.1	.74715	34.97	66.95	46.53	42.54	1.544	1
65	8	52.48	90.00	-17.16	0.00000	2616.8	-76.22	24997.2	.82011	35.47	63.00	44.25	43.57	1.385	1
66	7	31.38	90.00	-17.57	0.00000	2743.3	-70.40	19278.0	.75414	35.02	64.38	55.08	41.91	1.456	1
67	7	29.36	90.00	-21.90	0.00000	2744.7	-67.47	18651.6	.72914	35.03	65.37	52.00	36.02	1.442	1
68	5	38.95	90.00	-21.15	0.00000	2652.9	-71.36	19931.2	.76788	34.83	66.57	49.48	42.62	1.417	1
69	5	33.58	90.00	-19.03	0.00000	2777.0	-70.53	18789.6	.74616	34.59	66.36	51.14	43.09	1.482	1
70	7	31.77	90.00	-16.82	0.00000	2624.9	-68.60	20567.1	.77257	35.31	64.56	53.97	43.96	1.364	1
71	5	41.39	90.00	-21.08	0.00000	2721.9	-70.57	21603.6	.77920	35.54	64.09	46.57	45.25	1.425	1
72	6	43.50	90.00	-19.19	0.00000	2746.7	-72.93	19964.9	.76455	35.19	66.16	45.14	44.31	1.466	1
73	7	29.10	90.00	-19.06	0.00000	2695.9	-65.08	20116.7	.74018	35.46	64.03	54.31	34.38	1.445	1
74	6	44.01	90.00	-22.78	0.00000	2832.0	-72.87	19804.9	.74931	34.81	64.34	45.04	38.26	1.520	1
75	6	36.30	90.00	-22.62	0.00000	2733.1	-67.97	22399.1	.76929	35.57	65.00	48.40	35.23	1.494	1
76	9	38.28	90.00	-15.70	0.00000	2462.0	-69.64	20342.6	.78793	35.46	64.38	51.35	50.74	1.283	1
77	9	23.29	90.00	-11.77	0.00000	2698.1	-74.37	20614.8	.77959	35.35	65.47	60.49	29.70	1.435	1
78	8	32.48	90.00	-20.64	0.00000	2609.7	-69.50	22361.6	.79359	35.44	63.26	53.62	43.67	1.352	1
79	5	37.59	90.00	-13.92	0.00000	2822.7	-69.87	19747.7	.75461	34.86	65.43	46.96	47.37	1.488	1
80	6	43.72	90.00	-20.05	0.00000	2749.8	-72.15	22732.4	.78789	35.21	64.52	45.83	42.29	1.470	1
81	5	39.22	90.00	-21.53	0.00000	2710.4	-69.74	19724.3	.74700	34.08	65.41	48.28	36.73	1.472	1
82	8	25.56	90.00	-17.84	0.00000	2670.8	-68.14	19593.9	.74553	35.39	64.62	55.99	30.95	1.432	1
83	6	39.39	90.00	-22.89	0.00000	2822.2	-70.62	19130.2	.73386	34.99	66.34	45.76	36.07	1.523	1
84	8	31.57	90.00	-20.08	0.00000	2512.3	-65.39	19182.2	.74001	35.54	65.02	50.97	34.30	1.363	1
85	7	28.61	90.00	-20.82	0.00000	2816.7	-66.54	21238.0	.75258	35.25	64.98	49.65	36.51	1.503	1
86	5	39.33	90.00	-20.35	0.00000	2790.0	-70.08	18589.7	.73157	35.03	65.87	45.60	40.63	1.494	1
87	5	45.22	90.00	-19.21	0.00000	2695.4	-72.29	20738.3	.77311	35.08	65.04	45.65	44.09	1.439	1
88	8	27.31	90.00	-16.33	0.00000	2756.4	-70.02	19444.3	.75394	33.72	63.90	56.61	39.62	1.414	1
89	8	44.10	90.00	-17.38	0.00000	2584.9	-70.71	22109.9	.79835	35.15	63.07	47.43	51.92	1.359	1
90	6	39.16	90.00	-23.50	0.00000	2933.0	-70.32	20139.7	.73963	34.79	65.22	45.39	37.18	1.603	1
91	7	49.25	90.00	-16.63	0.00000	2783.0	-76.24	22501.0	.79004	34.27	65.09	41.69	40.99	1.494	1
92	6	32.78	90.00	-22.71	0.00000	2734.5	-69.30	21980.3	.77469	34.60	64.32	51.15	37.41	1.487	1
93	11	55.25	90.00	-18.94	0.00000	2953.3	-82.10	18657.3	.74414	35.11	65.99	39.95	33.59	1.620	1
94	6	32.77	90.00	-21.45	0.00000	2771.0	-65.53	22044.9	.76008	34.66	64.04	49.45	37.95	1.481	1
95	9	30.12	90.00	-15.63	0.00000	2528.7	-70.11	18816.0	.75953	35.26	65.24	56.56	39.36	1.343	1
96	10	36.14	90.00	-19.25	0.00000	2470.0	-67.36	18609.1	.75724	34.29	65.31	50.46	46.45	1.271	1
97	8	29.19	90.00	-19.65	0.00000	2902.2	-67.62	21668.5	.75762	34.88	64.95	52.36	37.63	1.579	1
98	6	39.04	90.00	-17.92	0.00000	2700.5	-69.15	20627.5	.76817	34.49	64.60	52.92	42.83	1.464	1
99	9	48.79	90.00	-20.17	0.00000	2989.4	-76.63	19079.4	.74001	34.99	65.09	42.15	39.01	1.640	1
100	7	25.40	90.00	-15.75	0.00000	2793.1	-74.72	19916.3	.77001	35.27	66.42	57.97	32.73	1.486	1

TABLE III.- SAMPLE OUTPUT – Continued

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EXPECTED CONTROLS
 ALPHA  37.623807       BETA  90.000000      DELTA -19.276359      THDOT 0.                TBURN  2745.2364         TAN -71.337526

EXPECTED ACTUAL INITIAL CONIC
 SMA -4292.1380       ECC  2.1016092       INC  36.712273       PER  63.561584       NOD  52.278343       TAN  299.93776

EXPECTED FINAL CONIC
 SMA  20599.435       ECC  .76535022       INC  35.011366       PER  65.009488       NOD  49.089143       TAN  40.702225

EXPECTED TARGET VARIABLES
  4.88328202753 6E-05   4806.23951378        35.0113664277        65.0094876144        49.0891428478        40.7022246230

EXPECTED COMMANDED DELTA V   1.46970041343

EXPECTED DELIVERED DELTA V   1.46919455468

STANDARD DEVIATION FOR CONTROLS
 ALPHA  7.7169245       BETA 0.                 DELTA  2.1439425      THDOT 0.                TBURN  120.08024         TAN  3.5759588

STANDARD DEVIATION FOR ACTUAL INITIAL CONIC
 SMA  234.34169       ECC  .10176001       INC  1.5878806       PER  2.6831534       NOD  2.4226739       TAN  2.7828447

STANDARD DEVIATION FOR FINAL CONIC
 SMA  1634.7060       ECC 2.03170389E-02  INC  .39314470       PER  1.0138377       NOD  4.7674693       TAN  5.2362056

STANDARD DEVIATION FOR TARGET VARIABLES
  3.64991640208 6E-06   231.229189452        .393144695122        1.01383768417        4.76746933388        5.23620558985

STANDARD DEVIATION OF COMMANDED VELOCITY  8.483972707831E-02

STANDARD DEVIATION OF DELIVERED VELOCITY  8.461163417308E-02

MEANS OF ERRORS GENERATED FROM TRACK
  -.421910919754       -1.88332898824       -.553759243777      -1.339424052054E-03 -1.088088800790E-04 -4.135369819293E-04

RECONSTRUCTED TRACK MATRIX
   525.158402345       -112.705625356        59.4418249092        3.937878860587E-02  1.041714394663E-02 -8.390957815877E-03
  -112.705625356        474.637783465        27.1751308681        4.768540994889E-03 -8.177640752319E-02  3.062752075895E-02
   59.4418249092        27.1751308681        454.100899976       -6.406958967675E-03  7.762653643678E-02 -2.213989792846E-02
  3.937878860587E-02  4.768540994889E-03 -6.406958967675E-03  3.998741494666E-04 -7.253153607464E-06 -1.929199007145E-05
  1.041714394663E-02 -8.177640752319E-02  7.762653643678E-02 -7.253153607464E-06  3.502916311165E-04 -4.387006112452E-05
 -8.390957815877E-03  3.062752075895E-02 -2.213989792846E-02 -1.929199007145E-05 -4.387006112452E-05  4.197793329858E-04

MEANS OF ERRORS GENERATED FROM DEV
  -30.2473433221       -22.5300903603       -18.6108414120      -3.081234822774E-03  3.657384269675E-03 -9.390269382167E-04

RECONSTRUCTED DEV MATRIX
   39655.2890603        4543.60909262       -4152.86162875       -.151974989407       -1.49380266752       -1.14644845597
   4543.60909262        53179.2050534       -1730.69617108        .220292730906       7.233919251886E-02  -.778095754316
  -4152.86162875       -1730.69617108        36461.4565829      -2.289210011510E-02  -1.93437725463       -.608500057705
  -.151974989407        .220292730906      -2.289210011510E-02  1.688993404792E-03  1.004327758003E-04 -8.718423806883E-05
  -1.49380266752       7.233919251886E-02  -1.93437725463       1.004327758003E-04  1.439312247609E-03  9.144122964360E-05
  -1.14644845597       -.778095754316       -.608500057705      -8.718423806883E-05  9.144122964360E-05  1.312832283470E-03
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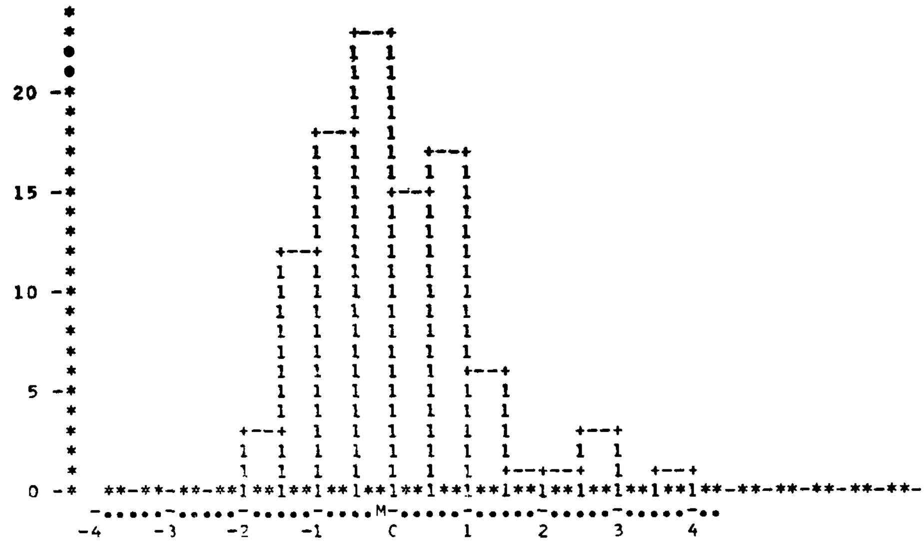
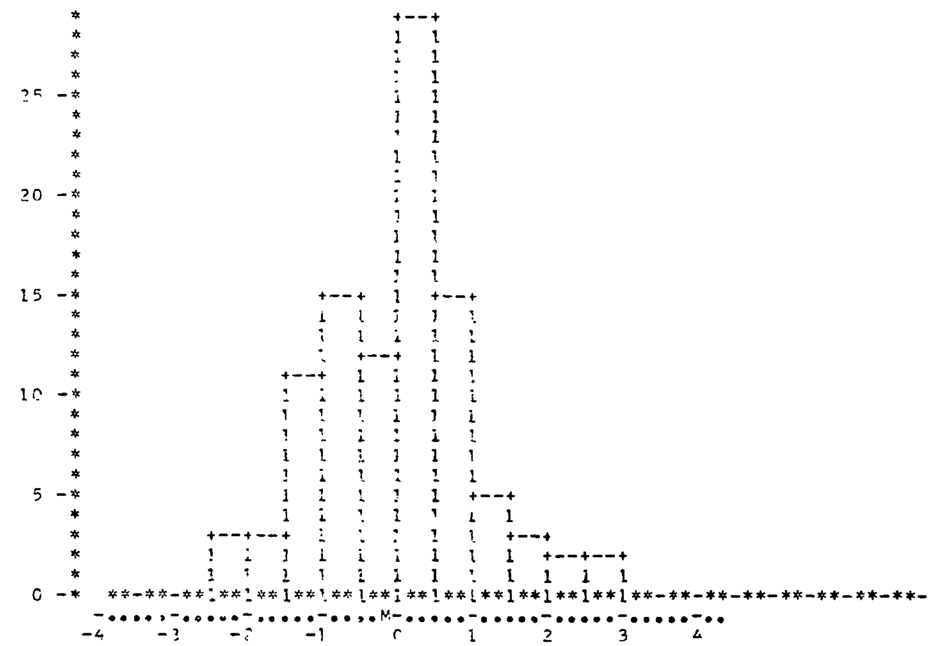


TOTAL N = 100

SEMI-MAJOR AXIS
SECOND CONIC

MEAN 20539.4353698 STD 1634.70598354

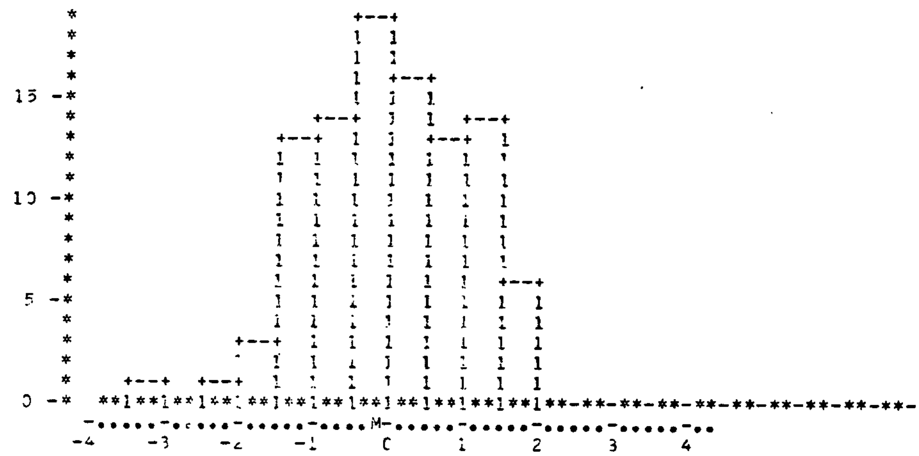


TOTAL N = 100

ECCENTRICITY
SECOND CONIC

MEAN .765350219756 STD 2.031703886048E-02

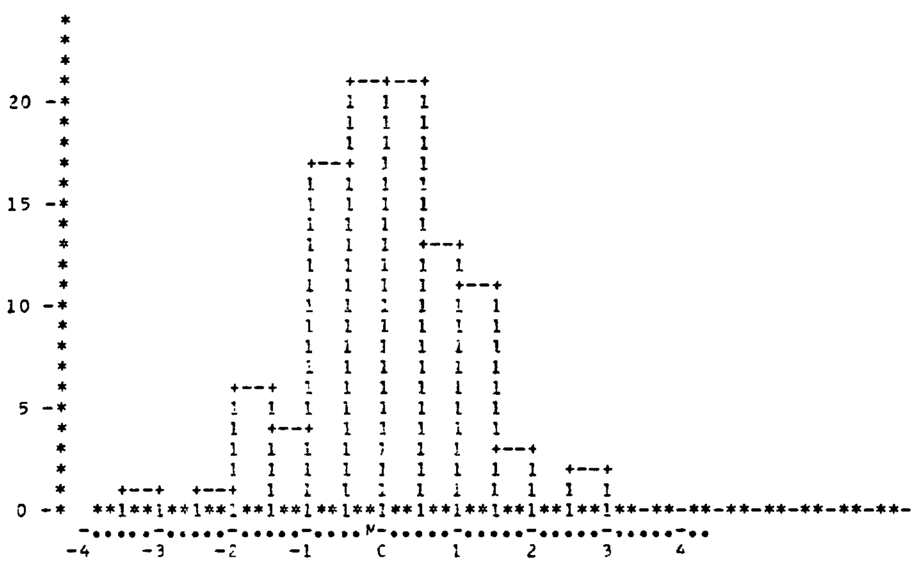
TABLE III.- SAMPLE OUTPUT – Continued



TOTAL N = 100

INCLINATION
SECOND CONIC

MEAN 35.9113564277 STD .393144695122

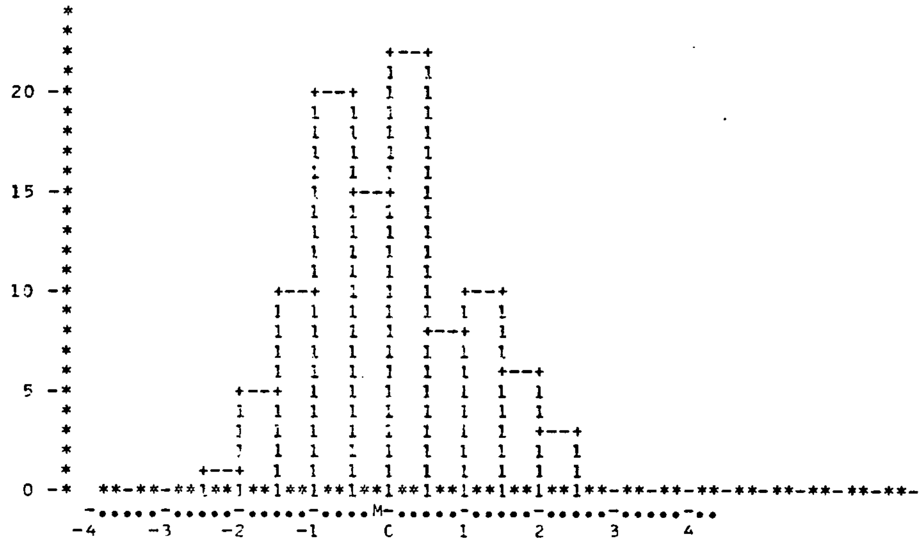


TOTAL N = 100

ARGUMENT OF PERIAPSIS
SECOND CONIC

MEAN 65.0094876144 STD 1.01383768417

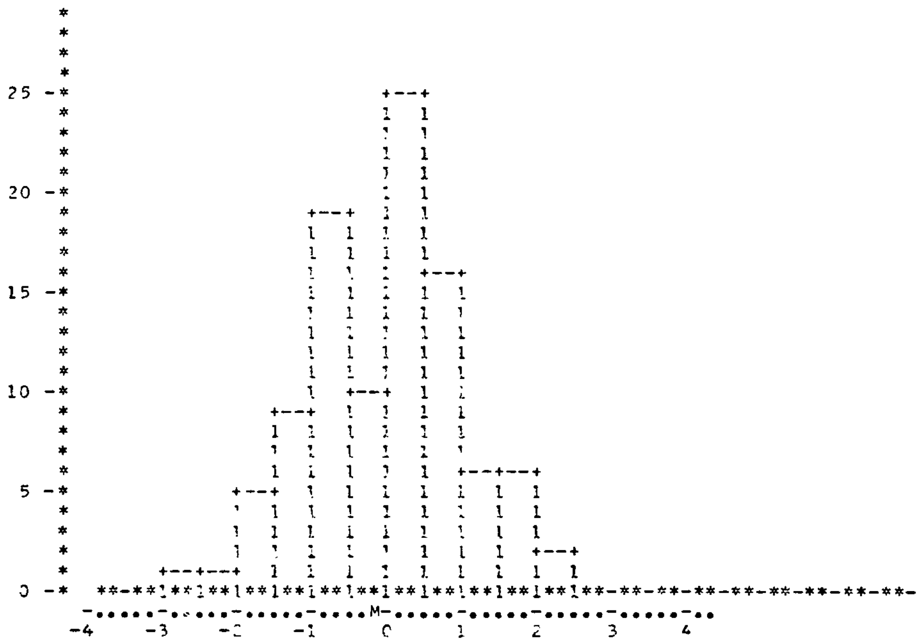
TABLE III.- SAMPLE OUTPUT – Continued



TOTAL N = 100

LONGITUDE OF ASCENDING NODE
SECOND CONIC

MEAN 49.0891428478 STD 4.76746933388

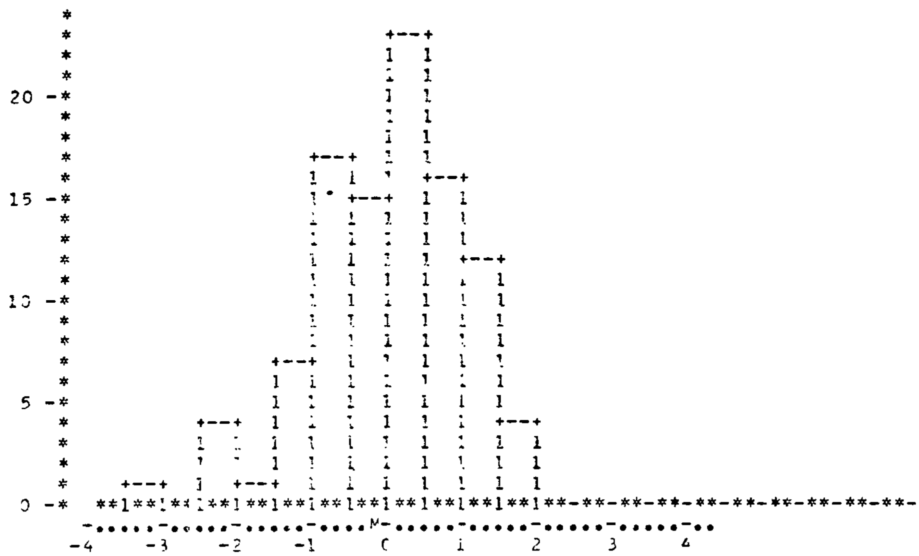


TOTAL N = 100

TRUE ANOMALY
SECOND CONIC

MEAN 40.7022246230 STD 5.23620558585

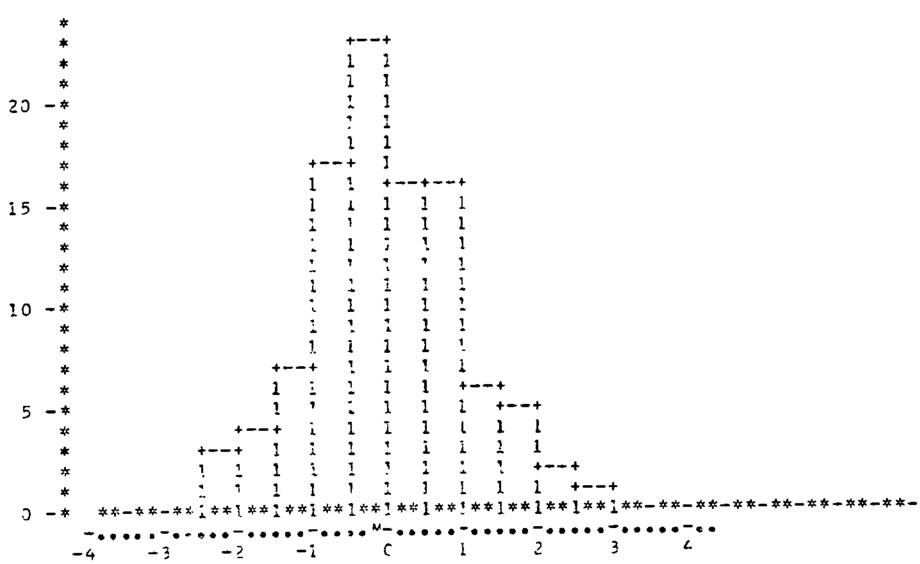
TABLE III.- SAMPLE OUTPUT – Continued



TOTAL N = 100

TARGET PARAMETER 1

MEAN 4.383282027536E-05 STD 3.6499164020866E-06

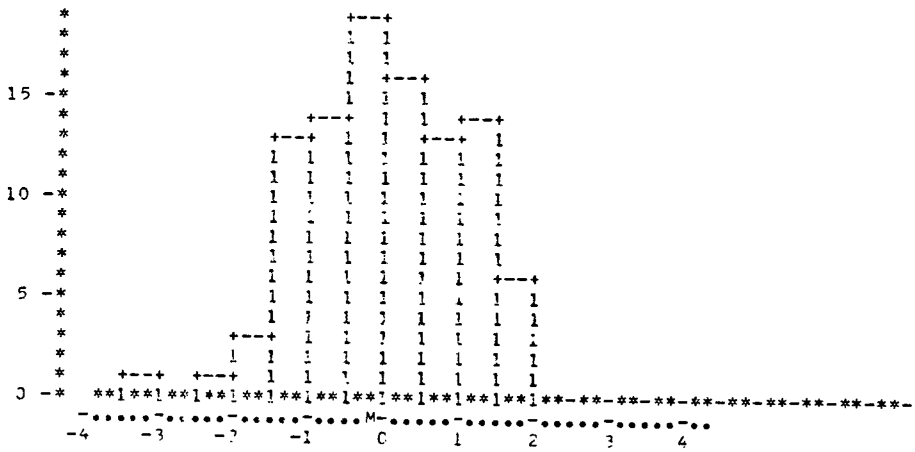


TOTAL N = 100

TARGET PARAMETER 2

MEAN 4806.23951378 STD 231.229189452

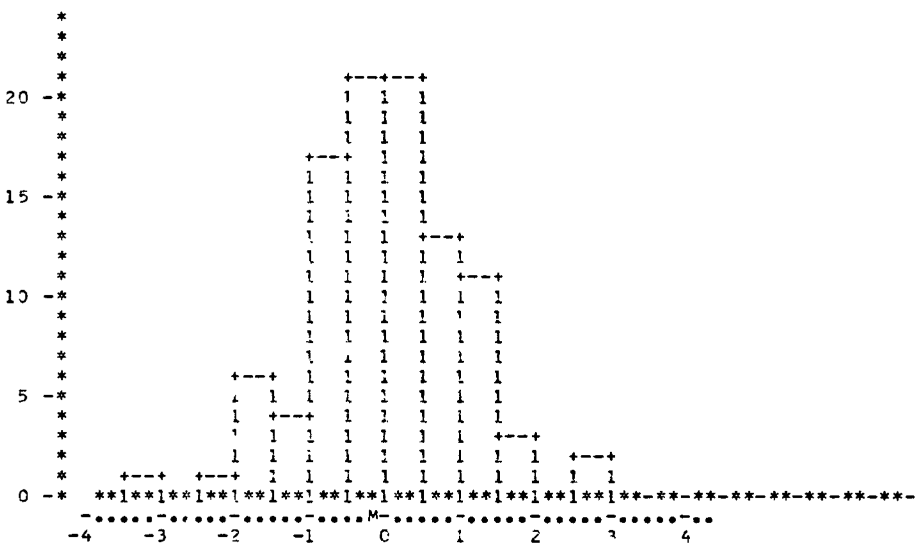
TABLE III.- SAMPLE OUTPUT – Continued



TOTAL N = 100

TARGET PARAMETER 3

MEAN 35.0113664277 STD .393144695122

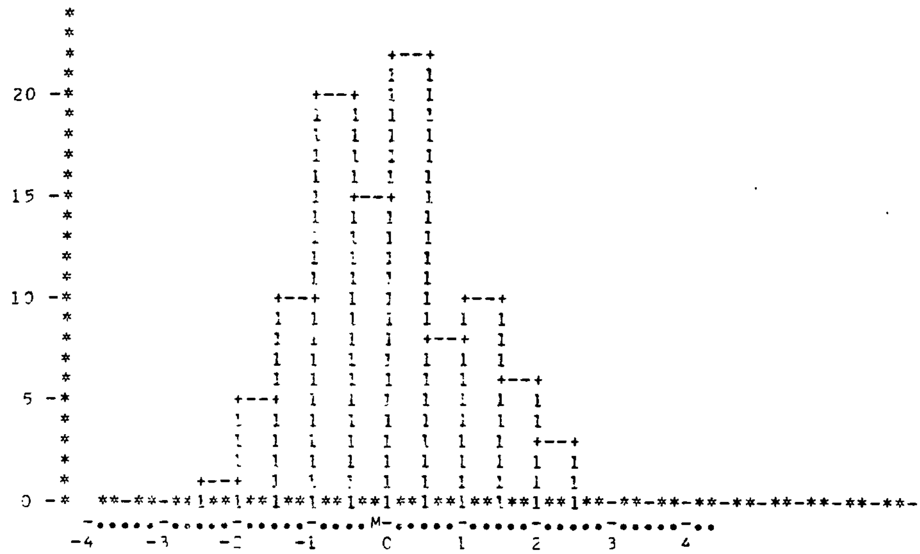


TOTAL N = 100

TARGET PARAMETER 4

MEAN 65.0094376144 STD 1.01383768417

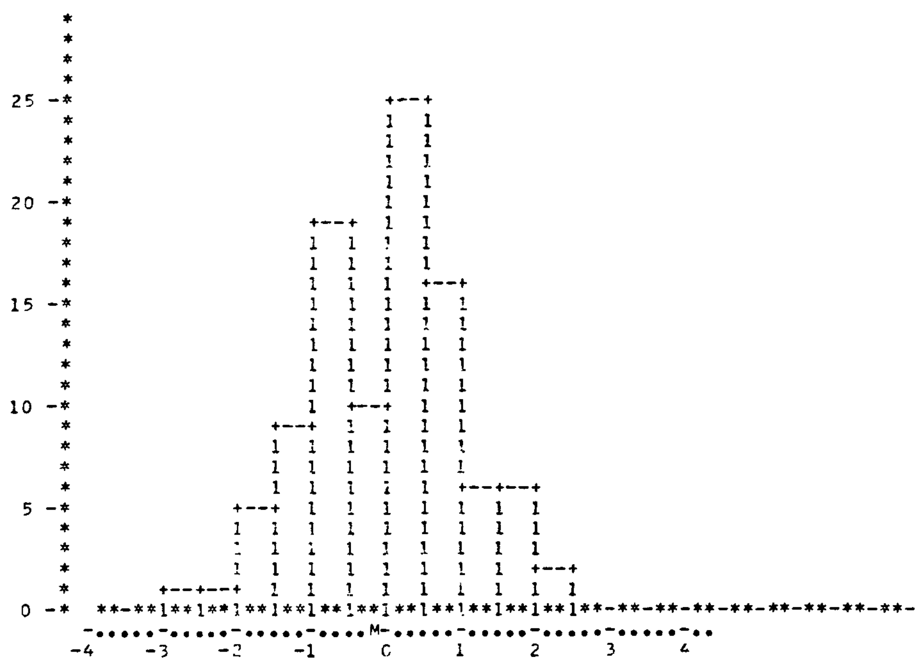
TABLE III.- SAMPLE OUTPUT – Continued



TOTAL N = 100

TARGET PARAMETER 5

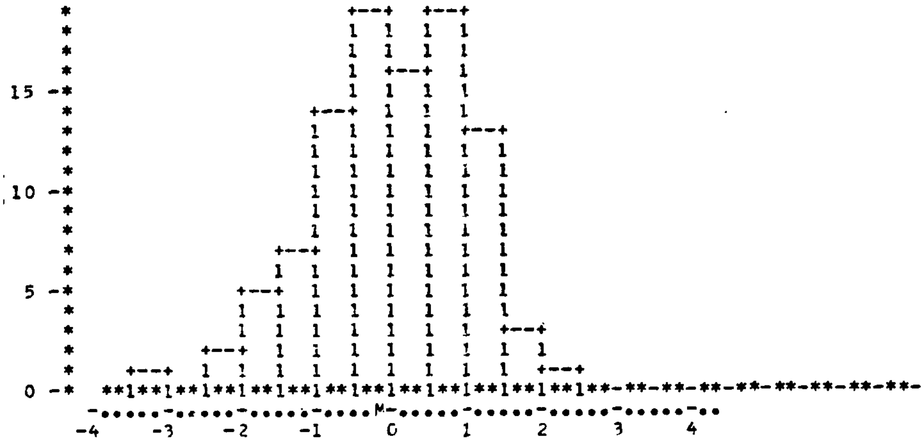
MEAN 49.0891428478 STD 4.76746933388



TOTAL N = 100

TARGET PARAMETER 6

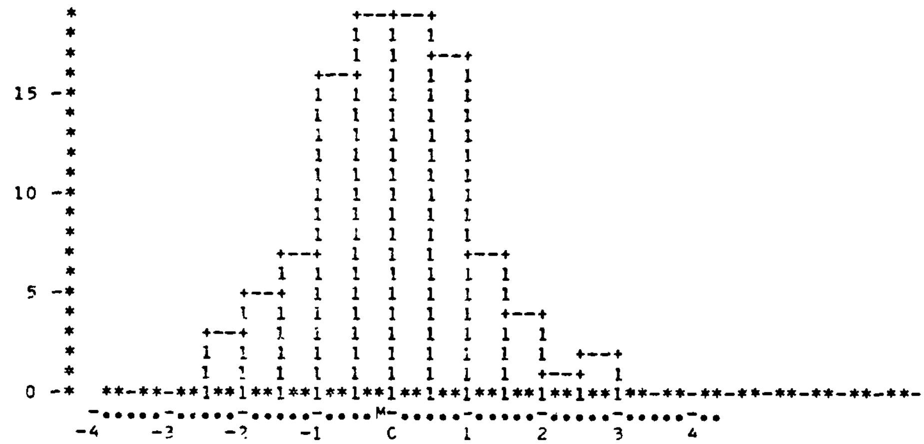
MEAN 40.7023246230 STD 5.23620558985



TOTAL N = 100

SEMI-MAJOR AXIS
INITIAL CONIC

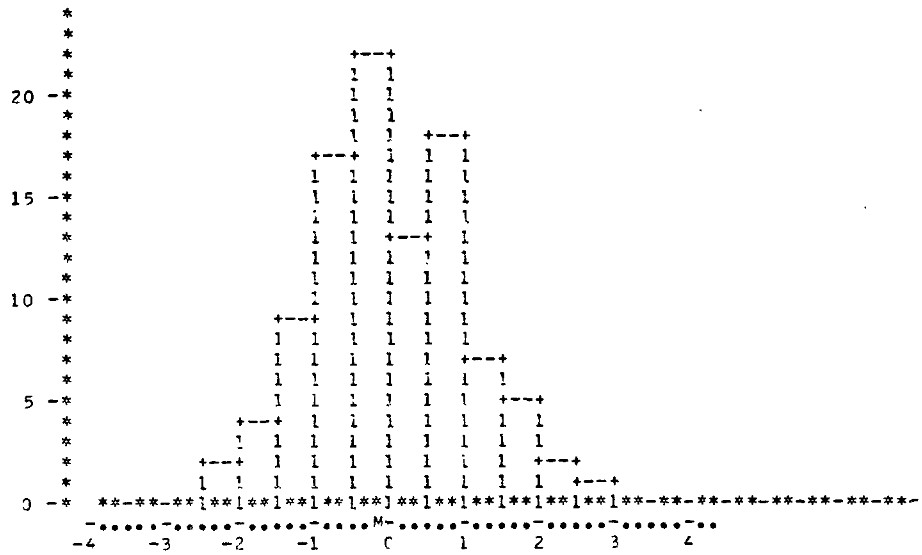
MEAN -4292.13796042 STD 234.341686470



TOTAL N = 100

ECCENTRICITY
INITIAL CONIC

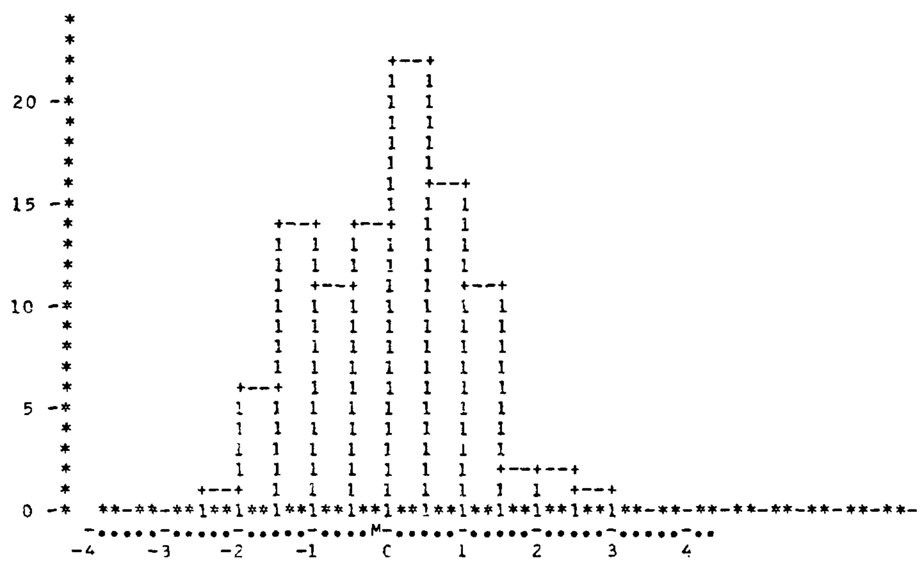
MEAN 2.10163917099 STD .101760005145



TOTAL N = 100

INCLINATION
INITIAL CONIC

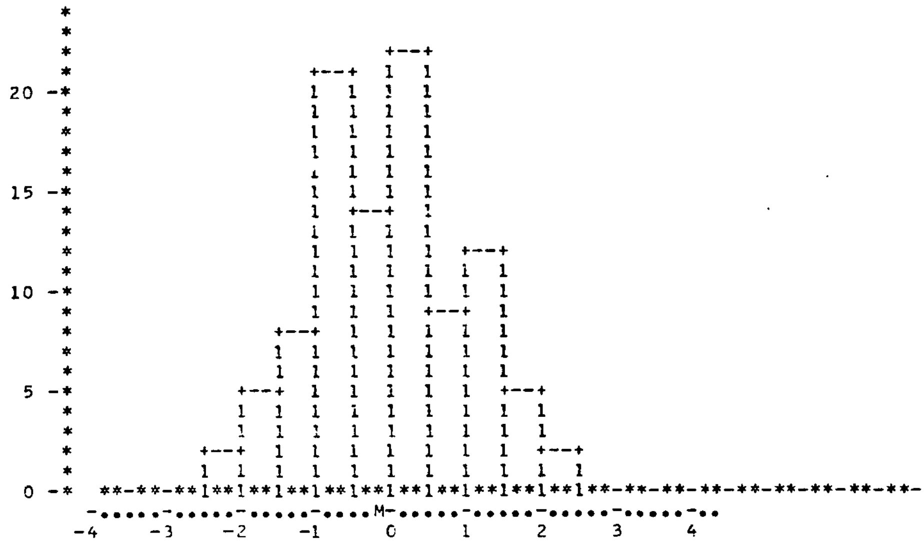
MEAN 36.7122732125 STD 1.58788059304



TOTAL N = 100

ARGUMENT OF PERIAPSIS
INITIAL CONIC

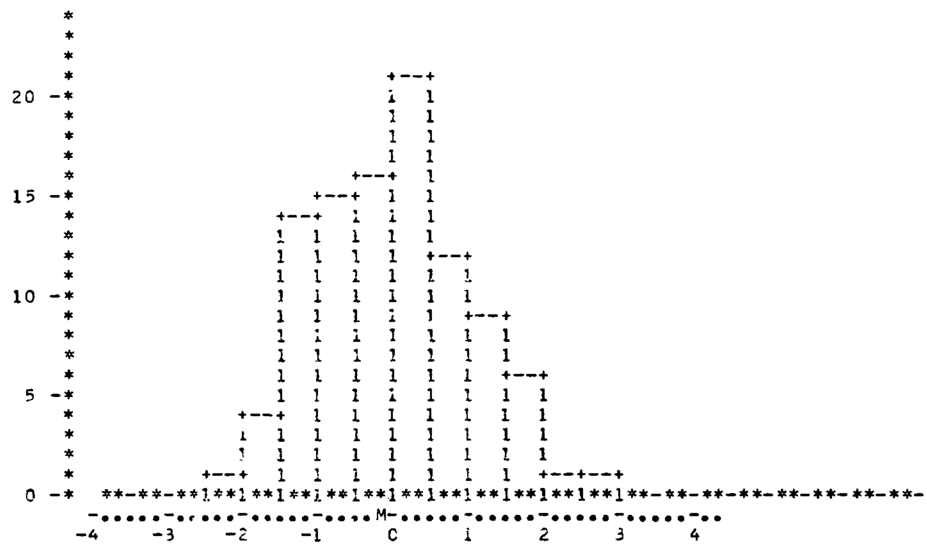
MEAN 63.5615838946 STD 2.68315335493



TOTAL N = 100

LONGITUDE OF ASCENDING NODE
INITIAL CONIC

MEAN 52.2783425755 STD 2.42267385815

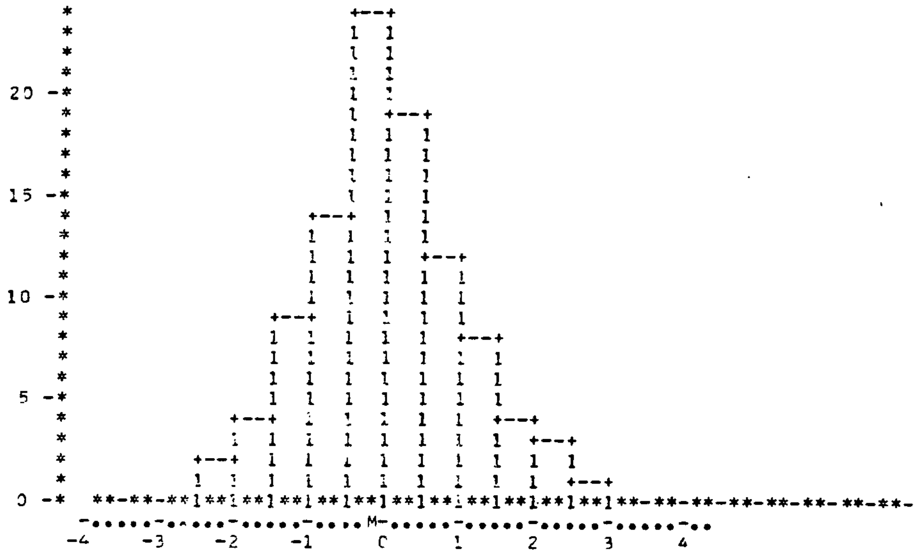


TOTAL N = 100

TRUE ANOMALY
INITIAL CONIC

MEAN 299.937764822 STD 2.78284472711

TABLE III.- SAMPLE OUTPUT – Continued



TOTAL N = 100

DELTA V

MEAN 1.46919456468 STD 8.461163417308E-02

TABLE III.- SAMPLE OUTPUT – Continued

CUMULATIVE PROBABILITY OF DELIVERED VELOCITY

PROBABILITY	VELOCITY		
1.000000000000E-02	1.27137620491	0.	1.000000000000+100
2.000000000000E-02	1.28250528350	.446641446712	1.33903513505
3.000000000000E-02	1.31722854155	1.04365184772	1.34810060397
4.000000000000E-02	1.31998490723	1.27557561650	1.35885593988
5.000000000000E-02	1.32630822293	1.28239789758	1.36284885340
6.000000000000E-02	1.33830965717	1.30489595068	1.36575117662
7.000000000000E-02	1.34280545126	1.31814115267	1.36912334027
8.000000000000E-02	1.35208127127	1.32025562947	1.37759791523
9.000000000000E-02	1.35944895499	1.32488931628	1.37973906722
1.000000000000E-01	1.36261347282	1.33262433186	1.38791217929
.110000000000	1.36365835024	1.33962431542	1.39497236692
.120000000000	1.36785674797	1.34347474696	1.40030451359
.130000000000	1.36955511379	1.35081972788	1.40312501698
.140000000000	1.37783224535	1.35699358438	1.40429012735
.150000000000	1.37857910285	1.36096597506	1.40920488651
.160000000000	1.38504038137	1.36292809119	1.41042831405
.170000000000	1.39276549130	1.36420917830	1.41357219305
.180000000000	1.39677603151	1.36772687910	1.41554921954
.190000000000	1.40169994270	1.36923954286	1.41989861506
.200000000000	1.40333425612	1.37506871218	1.42256645713
.210000000000	1.40420347628	1.37822395235	1.42498118430
.220000000000	1.40918756014	1.38109185701	1.42378316314
.230000000000	1.40930029980	1.38704181390	1.43221204877
.240000000000	1.41330634333	1.39330596774	1.43248687652
.250000000000	1.41396820297	1.39685333756	1.43386203514
.260000000000	1.41704121815	1.40121579045	1.43505702988
.270000000000	1.42164643892	1.40299511768	1.43762701935
.280000000000	1.42292420386	1.40393225468	1.43896457471
.290000000000	1.42545279611	1.40713393194	1.43999153974
.300000000000	1.42914837215	1.40924306411	1.44103601687
.310000000000	1.43226389398	1.41090637313	1.44245389895
.320000000000	1.43240492194	1.41351391810	1.44402512374
.330000000000	1.43376451763	1.41467605891	1.44703697893
.340000000000	1.43450243051	1.41773652414	1.44883702000
.350000000000	1.43728686953	1.42174283936	1.44893976555
.360000000000	1.43858748586	1.42293344126	1.44913658712
.370000000000	1.43976822459	1.42528979156	1.45081556949
.380000000000	1.44036719212	1.42867173845	1.45460866679
.390000000000	1.44195186036	1.43167214830	1.45713863088
.400000000000	1.44302651200	1.43237003171	1.45771575732
.410000000000	1.44499130278	1.43335471402	1.46113389614
.420000000000	1.44874190852	1.43424266584	1.46290925071
.430000000000	1.44890624271	1.43617513228	1.46386877967
.440000000000	1.44896134606	1.43801108697	1.46580453615
.450000000000	1.44923762882	1.43919701695	1.46916078022
.460000000000	1.45164086535	1.44005507913	1.47182671897
.470000000000	1.45603534749	1.44107215718	1.47340218926
.480000000000	1.45763290311	1.44239683722	1.47381362744
.490000000000	1.45775087315	1.44378591113	1.47768095555
.500000000000	1.46251019556	1.44634966492	1.48044287154

TABLE III.- SAMPLE OUTPUT – Continued

.510000000000	1.46306764924	1.44879795718	1.48182530070
.520000000000	1.46418413010	1.44892405129	1.48445040429
.530000000000	1.46644746605	1.44904658899	1.48624471460
.540000000000	1.47026344787	1.44995168567	1.48730667223
.550000000000	1.47248755124	1.45291069669	1.48869360772
.560000000000	1.47381030535	1.45648915387	1.49140353371
.570000000000	1.47381521376	1.45766622379	1.49393518387
.580000000000	1.47968186617	1.45910321810	1.49416550348
.590000000000	1.48087498510	1.46267139984	1.49564115393
.600000000000	1.48242400141	1.46339989455	1.49867974248
.610000000000	1.48538241715	1.46488438999	1.50048227000
.620000000000	1.48653504781	1.46768633503	1.50357709684
.630000000000	1.48801585118	1.47102722758	1.50538572821
.640000000000	1.48941683627	1.47297130681	1.50686897679
.650000000000	1.49389972334	1.47381222767	1.51293088754
.660000000000	1.49398853721	1.47628643160	1.51560279572
.670000000000	1.49449124487	1.48022422756	1.51930797355
.680000000000	1.49830627080	1.48163670849	1.52064710908
.690000000000	1.49981586092	1.48426676971	1.52149310233
.700000000000	1.50332448310	1.48625959865	1.52248951573
.710000000000	1.50528312196	1.48746375811	1.52754794185
.720000000000	1.50687479144	1.48896940803	1.53600323244
.730000000000	1.51342507061	1.49272731517	1.54048619482
.740000000000	1.51599006383	1.49397084743	1.54123572681
.750000000000	1.52030097312	1.49442481047	1.54387075817
.760000000000	1.52080526775	1.49807630623	1.54649633697
.770000000000	1.52195340504	1.49987424769	1.54792415792
.780000000000	1.52300940574	1.50351804225	1.55169955121
.790000000000	1.53402427423	1.50557998680	1.55568624486
.800000000000	1.54035650642	1.50870872887	1.56022939964
.810000000000	1.54094328666	1.51439856495	1.56671013745
.820000000000	1.54391745296	1.51808308006	1.57295081408
.830000000000	1.54689800856	1.52060270730	1.57524487589
.840000000000	1.54828297771	1.52163011469	1.57781101291
.850000000000	1.55387378353	1.52284711455	1.58318712240
.860000000000	1.55768526719	1.53383277460	1.58770302128
.870000000000	1.56530526584	1.54044035089	1.58984076881
.880000000000	1.57286612034	1.54178616097	1.59714954377
.890000000000	1.57560409735	1.54525790146	1.60169612197
.900000000000	1.57876182093	1.54776811753	1.60376158649
.910000000000	1.58726195110	1.55287009122	1.60772957590
.920000000000	1.58858544401	1.55790099351	1.61901251365
.930000000000	1.59781554028	1.56722745290	1.63296599211
.940000000000	1.60329983805	1.57423926465	1.64921689543
.950000000000	1.60427455565	1.57805047740	1.66681280736
.960000000000	1.61967171707	1.58736847729	1.66924670492
.970000000000	1.63954631657	1.59254648922	1.67920987569
.980000000000	1.66677449666	1.60241484718	1.70140202717
.990000000000	1.67074487504	1.60960259948	1.71595968099
1.00000000000	1.71830680926	0.	1.00000000000+100

TABLE III.- SAMPLE OUTPUT – Continued

PRCB	SMA	ECC	INC	ARGP	NODE	TA	1	2	3	4	5	6
.0100	17959.50	.7163	33.7206	61.8031	39.4157	25.2653	3.778E-05	4.314E+03	3.372E+01	6.180E+01	3.942E+01	2.527E+01
.0200	18020.91	.7193	34.0783	62.7742	39.9463	29.7020	3.972E-05	4.333E+03	3.408E+01	6.277E+01	3.995E+01	2.970E+01
.0300	18141.82	.7225	34.2657	62.9996	40.6845	30.2555	4.000E-05	4.338E+03	3.427E+01	6.300E+01	4.068E+01	3.026E+01
.0400	18184.68	.7291	34.2851	63.0726	41.2947	30.9538	4.001E-05	4.389E+03	3.429E+01	6.307E+01	4.129E+01	3.095E+01
.0500	18459.65	.7316	34.3170	63.2624	41.3274	31.0130	4.070E-05	4.405E+03	3.432E+01	6.326E+01	4.133E+01	3.101E+01
.0600	18573.10	.7339	34.4621	63.3762	41.6851	31.4585	4.262E-05	4.433E+03	3.446E+01	6.338E+01	4.169E+01	3.146E+01
.0700	18589.65	.7365	34.4729	63.3917	42.0453	32.7321	4.392E-05	4.458E+03	3.447E+01	6.339E+01	4.205E+01	3.273E+01
.0800	18609.15	.7396	34.4767	63.4468	42.0468	33.5890	4.399E-05	4.497E+03	3.448E+01	6.345E+01	4.205E+01	3.359E+01
.0900	18651.58	.7400	34.4386	63.5518	42.1527	34.2980	4.421E-05	4.503E+03	3.449E+01	6.355E+01	4.215E+01	3.430E+01
.1000	18657.27	.7400	34.5007	63.6644	43.3115	34.3814	4.444E-05	4.518E+03	3.450E+01	6.356E+01	4.331E+01	3.438E+01
.1100	18705.50	.7402	34.5326	63.7747	43.3897	34.3823	4.464E-05	4.525E+03	3.453E+01	6.377E+01	4.339E+01	3.438E+01
.1200	18789.64	.7414	34.5375	63.9016	43.7581	34.5038	4.472E-05	4.528E+03	3.454E+01	6.390E+01	4.376E+01	3.450E+01
.1300	18813.35	.7418	34.5549	64.0293	43.8224	35.2253	4.515E-05	4.544E+03	3.455E+01	6.403E+01	4.382E+01	3.523E+01
.1400	18814.71	.7428	34.5943	64.0406	43.9604	35.2507	4.523E-05	4.567E+03	3.459E+01	6.404E+01	4.396E+01	3.525E+01
.1500	18915.98	.7431	34.5965	64.0910	44.2549	35.2886	4.535E-05	4.581E+03	3.460E+01	6.409E+01	4.425E+01	3.529E+01
.1600	19050.14	.7441	34.5994	64.0931	44.2747	35.3494	4.536E-05	4.581E+03	3.460E+01	6.409E+01	4.427E+01	3.535E+01
.1700	19079.39	.7441	34.6001	64.1059	44.7738	35.6721	4.550E-05	4.582E+03	3.460E+01	6.411E+01	4.477E+01	3.567E+01
.1800	19117.82	.7455	34.6053	64.1466	44.8114	35.9644	4.560E-05	4.590E+03	3.461E+01	6.415E+01	4.481E+01	3.596E+01
.1900	19125.04	.7462	34.6388	64.2013	44.9004	36.0192	4.570E-05	4.598E+03	3.464E+01	6.420E+01	4.490E+01	3.602E+01
.2000	19130.22	.7466	34.6573	64.2041	44.9747	36.0746	4.578E-05	4.607E+03	3.466E+01	6.420E+01	4.497E+01	3.607E+01
.2100	19182.22	.7470	34.6627	64.3187	45.0437	36.1880	4.591E-05	4.615E+03	3.466E+01	6.432E+01	4.504E+01	3.619E+01
.2200	19277.96	.7471	34.6678	64.3326	45.1450	36.4011	4.606E-05	4.615E+03	3.467E+01	6.433E+01	4.514E+01	3.640E+01
.2300	19389.49	.7473	34.6822	64.3403	45.1467	36.4059	4.609E-05	4.616E+03	3.469E+01	6.434E+01	4.515E+01	3.641E+01
.2400	19444.33	.7493	34.6904	64.3773	45.3878	36.5147	4.610E-05	4.619E+03	3.469E+01	6.438E+01	4.539E+01	3.651E+01
.2500	19465.65	.7500	34.6981	64.3797	45.4015	36.6687	4.615E-05	4.626E+03	3.470E+01	6.438E+01	4.540E+01	3.667E+01
.2600	19476.99	.7514	34.7312	64.4163	45.4401	36.7262	4.629E-05	4.628E+03	3.475E+01	6.442E+01	4.544E+01	3.673E+01
.2700	19482.10	.7520	34.7554	64.4669	45.4814	37.1729	4.634E-05	4.640E+03	3.476E+01	6.447E+01	4.548E+01	3.717E+01
.2800	19593.92	.7524	34.7686	64.4703	45.6044	37.1763	4.639E-05	4.677E+03	3.477E+01	6.447E+01	4.560E+01	3.718E+01
.2900	19598.24	.7526	34.7885	64.4879	45.6471	37.4084	4.655E-05	4.681E+03	3.479E+01	6.449E+01	4.565E+01	3.741E+01
.3000	19633.63	.7539	34.8053	64.5250	45.7117	37.6298	4.671E-05	4.689E+03	3.481E+01	6.452E+01	4.571E+01	3.763E+01
.3100	19643.34	.7541	34.8083	64.5532	45.7422	37.7200	4.709E-05	4.690E+03	3.481E+01	6.455E+01	4.574E+01	3.772E+01
.3200	19724.27	.7546	34.8093	64.5622	45.7620	37.7647	4.745E-05	4.695E+03	3.481E+01	6.456E+01	4.576E+01	3.776E+01
.3300	19747.73	.7572	34.8307	64.5977	45.8350	37.8615	4.768E-05	4.701E+03	3.483E+01	6.460E+01	4.583E+01	3.786E+01
.3400	19804.92	.7576	34.8479	64.6169	46.3642	37.9523	4.771E-05	4.705E+03	3.485E+01	6.462E+01	4.636E+01	3.795E+01
.3500	19862.73	.7578	34.8625	64.6267	46.5345	38.0173	4.774E-05	4.715E+03	3.486E+01	6.463E+01	4.653E+01	3.802E+01
.3600	19916.35	.7579	34.8762	64.6618	46.5707	38.2408	4.779E-05	4.717E+03	3.488E+01	6.466E+01	4.657E+01	3.824E+01
.3700	19931.22	.7581	34.8318	64.6829	46.7690	38.2553	4.784E-05	4.719E+03	3.488E+01	6.468E+01	4.677E+01	3.826E+01
.3800	19964.14	.7592	34.8834	64.6840	46.7725	39.0117	4.792E-05	4.722E+03	3.488E+01	6.468E+01	4.677E+01	3.901E+01
.3900	19964.93	.7595	34.9002	64.6850	46.9568	39.3506	4.814E-05	4.724E+03	3.490E+01	6.469E+01	4.696E+01	3.935E+01
.4000	19975.65	.7599	34.9215	64.6941	47.4345	39.3551	4.822E-05	4.729E+03	3.492E+01	6.469E+01	4.743E+01	3.936E+01
.4100	20030.71	.7601	34.9317	64.7579	47.5944	39.4867	4.846E-05	4.740E+03	3.493E+01	6.476E+01	4.759E+01	3.949E+01
.4200	20116.71	.7608	34.9508	64.7652	47.8111	39.6218	4.848E-05	4.759E+03	3.496E+01	6.477E+01	4.781E+01	3.962E+01
.4300	20131.47	.7621	34.9544	64.7785	48.1456	40.3164	4.851E-05	4.770E+03	3.496E+01	6.478E+01	4.815E+01	4.032E+01
.4400	20139.67	.7645	34.9724	64.8794	48.1964	40.3994	4.852E-05	4.770E+03	3.497E+01	6.488E+01	4.820E+01	4.040E+01
.4500	20205.61	.7656	34.9329	64.9054	48.2755	40.6261	4.862E-05	4.774E+03	3.498E+01	6.491E+01	4.828E+01	4.063E+01
.4600	20246.69	.7655	34.9893	64.9195	48.3018	40.9436	4.888E-05	4.782E+03	3.499E+01	6.492E+01	4.830E+01	4.094E+01
.4700	20249.15	.7666	34.9894	64.9495	48.3299	40.9891	4.892E-05	4.784E+03	3.499E+01	6.495E+01	4.833E+01	4.099E+01
.4800	20256.74	.7658	34.9904	64.9539	48.4048	41.0153	4.904E-05	4.788E+03	3.499E+01	6.495E+01	4.840E+01	4.102E+01
.4900	20281.73	.7676	34.9920	64.9805	48.5162	41.2520	4.916E-05	4.792E+03	3.499E+01	6.498E+01	4.852E+01	4.125E+01
.5000	20282.16	.7679	35.0051	65.0048	48.7771	41.3566	4.920E-05	4.794E+03	3.501E+01	6.500E+01	4.878E+01	4.136E+01

TABLE III.- SAMPLE OUTPUT – Concluded

.5100	20325.43	.7682	35.0111	65.0207	48.9386	41.3709	4.930E-05	4.797E+03	3.501E+01	6.502E+01	4.894E+01	4.137E+01
.5200	20342.60	.7682	35.0239	65.0360	49.4466	41.4382	4.931E-05	4.802E+03	3.502E+01	6.504E+01	4.945E+01	4.144E+01
.5300	20392.36	.7687	35.0285	65.0888	49.4754	41.6255	4.937E-05	4.803E+03	3.503E+01	6.509E+01	4.948E+01	4.163E+01
.5400	20443.42	.7688	35.0309	65.0897	49.4781	41.7498	4.938E-05	4.806E+03	3.503E+01	6.509E+01	4.948E+01	4.175E+01
.5500	20453.19	.7693	35.0397	65.0901	49.6502	41.7630	4.939E-05	4.817E+03	3.504E+01	6.509E+01	4.965E+01	4.176E+01
.5600	20567.06	.7694	35.0568	65.0940	49.9076	41.9083	4.949E-05	4.822E+03	3.507E+01	6.509E+01	4.991E+01	4.191E+01
.5700	20609.51	.7700	35.0767	65.1084	49.9366	42.2939	4.965E-05	4.827E+03	3.508E+01	6.511E+01	4.994E+01	4.229E+01
.5800	20614.81	.7703	35.1066	65.1998	49.9737	42.4465	4.967E-05	4.832E+03	3.511E+01	6.520E+01	4.997E+01	4.245E+01
.5900	20627.54	.7707	35.1094	65.2085	50.1443	42.4562	4.971E-05	4.846E+03	3.511E+01	6.521E+01	5.014E+01	4.246E+01
.6000	20634.70	.7708	35.1222	65.2173	50.2390	42.5089	4.992E-05	4.858E+03	3.512E+01	6.522E+01	5.024E+01	4.251E+01
.6100	20738.32	.7721	35.1353	65.2188	50.2370	42.5362	5.006E-05	4.859E+03	3.514E+01	6.522E+01	5.034E+01	4.254E+01
.6200	20771.64	.7726	35.1548	65.2395	50.4117	42.5571	5.009E-05	4.865E+03	3.515E+01	6.524E+01	5.041E+01	4.256E+01
.6300	20867.07	.7728	35.1762	65.3074	50.4586	42.6173	5.009E-05	4.865E+03	3.518E+01	6.531E+01	5.046E+01	4.262E+01
.6400	20902.99	.7730	35.1767	65.3285	50.4975	42.7145	5.017E-05	4.873E+03	3.518E+01	6.533E+01	5.050E+01	4.271E+01
.6500	20923.34	.7731	35.1778	65.3739	50.8142	42.8295	5.021E-05	4.875E+03	3.518E+01	6.537E+01	5.081E+01	4.283E+01
.6600	20946.96	.7731	35.1953	65.3987	50.8679	42.8510	5.035E-05	4.877E+03	3.519E+01	6.540E+01	5.087E+01	4.285E+01
.6700	20958.46	.7734	35.2079	65.4103	50.9720	43.0323	5.049E-05	4.907E+03	3.521E+01	6.541E+01	5.097E+01	4.303E+01
.6800	20973.91	.7736	35.2281	65.4253	51.0506	43.0889	5.064E-05	4.909E+03	3.523E+01	6.543E+01	5.105E+01	4.309E+01
.6900	21076.48	.7738	35.2440	65.4308	51.1405	43.2985	5.070E-05	4.917E+03	3.524E+01	6.543E+01	5.114E+01	4.330E+01
.7000	21238.02	.7738	35.2489	65.4665	51.1463	43.2990	5.091E-05	4.919E+03	3.525E+01	6.547E+01	5.115E+01	4.330E+01
.7100	21430.04	.7740	35.2590	65.5145	51.3315	43.5703	5.093E-05	4.936E+03	3.526E+01	6.551E+01	5.133E+01	4.357E+01
.7200	21481.65	.7747	35.2694	65.5305	51.3512	43.6085	5.102E-05	4.941E+03	3.527E+01	6.553E+01	5.135E+01	4.361E+01
.7300	21557.33	.7747	35.2887	65.5587	51.4234	43.6685	5.104E-05	4.952E+03	3.529E+01	6.556E+01	5.142E+01	4.367E+01
.7400	21578.27	.7757	35.3132	65.5637	51.9994	43.7537	5.133E-05	4.954E+03	3.531E+01	6.556E+01	5.200E+01	4.375E+01
.7500	21603.57	.7761	35.3242	65.5813	52.1941	43.8263	5.134E-05	4.957E+03	3.532E+01	6.558E+01	5.219E+01	4.383E+01
.7600	21668.47	.7771	35.3276	65.5941	52.2220	43.9649	5.137E-05	4.960E+03	3.533E+01	6.559E+01	5.222E+01	4.396E+01
.7700	21692.47	.7774	35.3331	65.6293	52.3561	44.0916	5.143E-05	4.965E+03	3.533E+01	6.563E+01	5.236E+01	4.409E+01
.7800	21697.39	.7792	35.3467	65.6567	52.4686	44.3143	5.157E-05	4.982E+03	3.535E+01	6.566E+01	5.247E+01	4.431E+01
.7900	21710.64	.7796	35.3472	65.6690	52.6924	44.8445	5.187E-05	4.983E+03	3.535E+01	6.567E+01	5.269E+01	4.484E+01
.8000	21780.42	.7799	35.3905	65.8725	52.9177	44.8562	5.213E-05	4.986E+03	3.539E+01	6.587E+01	5.292E+01	4.486E+01
.8100	21841.73	.7815	35.4078	65.9752	53.6217	44.9562	5.227E-05	4.987E+03	3.541E+01	6.598E+01	5.362E+01	4.496E+01
.8200	21879.81	.7823	35.4145	65.9876	53.9690	45.0106	5.229E-05	4.990E+03	3.541E+01	6.599E+01	5.397E+01	4.501E+01
.8300	21929.67	.7828	35.4223	65.9889	54.3090	45.0521	5.231E-05	4.990E+03	3.542E+01	6.599E+01	5.431E+01	4.505E+01
.8400	21980.29	.7829	35.4377	66.0218	54.7639	45.2284	5.241E-05	5.014E+03	3.544E+01	6.602E+01	5.476E+01	4.523E+01
.8500	22044.85	.7834	35.4553	66.0397	54.8574	45.2520	5.249E-05	5.014E+03	3.546E+01	6.604E+01	5.486E+01	4.525E+01
.8600	22049.52	.7840	35.4576	66.1011	55.0630	45.5357	5.315E-05	5.015E+03	3.546E+01	6.610E+01	5.506E+01	4.554E+01
.8700	22109.92	.7840	35.4584	66.1259	55.0777	46.0531	5.315E-05	5.052E+03	3.546E+01	6.613E+01	5.508E+01	4.605E+01
.8800	22149.74	.7842	35.4596	66.1636	55.1636	46.3540	5.315E-05	5.061E+03	3.546E+01	6.616E+01	5.516E+01	4.635E+01
.8900	22361.64	.7866	35.4680	66.2794	55.3793	46.4542	5.322E-05	5.091E+03	3.547E+01	6.628E+01	5.538E+01	4.645E+01
.9000	22399.08	.7879	35.5066	66.3287	55.9927	47.3672	5.346E-05	5.105E+03	3.551E+01	6.633E+01	5.599E+01	4.737E+01
.9100	22501.01	.7879	35.5382	66.3354	56.0377	47.3681	5.360E-05	5.113E+03	3.554E+01	6.634E+01	5.604E+01	4.737E+01
.9200	22620.09	.7900	35.5440	66.3553	56.4434	47.6594	5.361E-05	5.116E+03	3.554E+01	6.636E+01	5.644E+01	4.766E+01
.9300	22732.44	.7936	35.5510	66.3906	56.5616	49.2510	5.374E-05	5.168E+03	3.556E+01	6.639E+01	5.656E+01	4.925E+01
.9400	22767.40	.7968	35.5700	66.4176	56.6055	49.6666	5.379E-05	5.227E+03	3.557E+01	6.642E+01	5.661E+01	4.967E+01
.9500	23463.86	.7984	35.6170	66.4886	57.1089	49.9788	5.384E-05	5.244E+03	3.562E+01	6.649E+01	5.711E+01	4.998E+01
.9600	24572.03	.8045	35.6237	66.5745	57.6279	50.1261	5.417E-05	5.252E+03	3.562E+01	6.657E+01	5.763E+01	5.013E+01
.9700	24996.00	.8094	35.6383	66.9452	57.9681	50.4681	5.499E-05	5.255E+03	3.564E+01	6.695E+01	5.797E+01	5.047E+01
.9800	24997.24	.8111	35.6588	66.9858	58.9622	50.7445	5.512E-05	5.289E+03	3.566E+01	6.699E+01	5.896E+01	5.074E+01
.9900	25177.67	.8170	35.7272	67.5607	59.1320	51.6823	5.549E-05	5.379E+03	3.573E+01	6.756E+01	5.913E+01	5.168E+01
1.0000	26466.95	.8201	35.7657	67.6548	60.4932	51.9226	5.568E-05	5.399E+03	3.577E+01	6.765E+01	6.049E+01	5.192E+01

TABLE IV.- HISTOGRAM SYMBOLS

Symbol	Interval value		
	2	4	8
+ - - +	Add 0	Add 0	Add 0
+ · · +	-----	Add 1	Add 1, 2, or 3
++++	Add 1	Add 2	Add 4
+ I I +	-----	Add 3	Add 5, 6, or 7

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